LOW FREQUENCY NONLINEAR DISTORTION OF BIPOLAR TRANSISTORS IN BASIC AMPLIFYING STAGES

by

CLAS-GÖRAN AGNVALL

DEPARTMENT OF PHYSICS
ELECTRONIC DIVISION
UNIVERSITY OF LUND

1982.

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Abstract

This thesis presents theoretical derivations and measurements of low frequency nonlinear distortion in bipolar transistor amplifiers. Two basic amplifying stages are analyzed: the common emitter series feedback stage and the differential pair with finite common emitter resistance.
For the differential pair the Ebers-Moll model is used as nonlinear device model. Compact algebraic expressions are presented for the second and third order distortions as functions of emitter series impedance and common emitter impedance.
A modification of the Integral Control Model is presented and applied to the low power transistor BC550C. The model includes nonlinear current gain and Early-effect. Second order distortion is derived for a common emitter feedback stage using this modified model.
The interaction between the different sources of distortion and the external emitter and collector resistances is investigated and a compact algebraic expressions is given for the result.
The performed measurements confirm the validity of the derived distortion expressions.

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Tryckbaren, Lund 1982

TO MOTHER
and
TO THE MEMORY OF
MY FATHER

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I INTRODUCTION

The nonlinear distortion produced by a transistor amplifier is often a major design constraint. The technique too often used in order to reduce the nonlinearities, is to apply negative feedback and more seldom of avoiding unnessesary sources of distortion by proper design.

The sources of the distortion are the nonlinear transfer functions of the individual transistor devices, and the interaction between these nonlinearities and the external impedances.

To analyze and understand the total distortion behaviour of an amplifier, it is therefore of vital importance to gain information on the nonlinear transfer function of the individual transistors and then to apply the proper analytic tools. Thus the analysis of amplifier distortion is deeply interrelated with the transistor device modeling.

There are a variety of bipolar transistor models for different purposes. A bibliography is presented in [12].

One of the most frequently used models is the Ebers-Moll model, which is simple and includes only the forward pn junction nonlinearity. Despite its simplicity it is able to predict the distortion properties in many applications. For example, the cancellation of the third and higher order distortion products in a common emitter current feedback stage has successfully been described by many authors using this model [14],[15],[16].

The differential amplifier with infinite common emitter impedance has also been the subject of distortion analysis, with the EM model as nonlinear device model [18], [20].

The main drawback of the basic EM model is that it fails to explain certain effects which are visible for high bias levels and for high drive and load impedances.

A transistor model that effectively describes several effects not accounted for by the EM model is the integral charge

control model (ICM), presented by Gummel and Poon 1970 [7]. Apart from high frequency effects it also accounts for the Early-effect, high injection and the decay of current gain for low collector currents.

Several authors have applied this model to high frequency amplifier distortion studies, e.g.[21].

It is the purpose of this thesis to review some of the basic theory for the analysis of low frequency distortion for the common emitter and the differential amplifier using the EM-model, and to extend the analysis of the differential amplifier to cover a finite common emitter impedance. For the differential amplifier compact algebraic expressions are presented for the distortion as a function of emitter series impedance and common emitter impedance. Furthermore, the purpose is to investigate the low frequency effects of a modified Gummel-Poon model applied to a common emitter feedback stage. A detailed analysis is given for the influence of the Early-effect, and, thereby, for the collector load impedance on the second order distortion. The effects of the nonlinear current gain and its dependency upon the Early-effect are also investigated.

A compact algebraic expression of the second order distortion as a function of transistor bias level and external base, collector and emitter impedances is given.

The mathematical analytic tool used in this text is the Taylor series expansion. Since the analysis covers feedback around nonlinear elements, the Taylor expansion is performed using implicite derivation or direct series reversal, which basicly is the same method.

The organisation of the text is, apart from this introduction, divided into six chapters. An introductory discussion of the properties of weakly nonlinear transfer functions is given in chapter two.

Chapter tree will focus on a description of the two transistor models used in this text.

To facilate the Taylor series expansion the original ICM is modified. This modified model, together with the algebraic distortion expression is described in chapter four.

The measurements are presented in chapters five and six. A brief description of model parameter measurements is presented in chapter five, and the distortion measurements are presented in chapter six.

II TRANSFER FUNCTIONS

Normally, the purpose of an electronic system is to transform and transfer energy in the shape of an electric signal from an input source to an output load. The influence which the system has on the signal, referred to the input and the output terminals can be stated, for example, in tabular or graphic form, but, more often, it is given as a mathematical function. Thus, the transfer function that is of interest can be defined as the ratio of the output to the input signal.

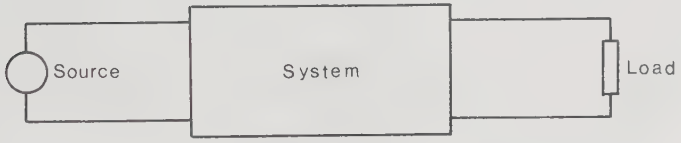

Fig.1. Schematic electronic system

The transfer function of any real electronic system is always frequency dependent and nonlinear. The frequency dependence arises because of the finite physical dimensions and thereto attached reactances. However, normally it is possible to confine the analysis of such a system to a band of frequencies for which the reactive effects may be neglected.

The nonlinear behavior arises mainly from active elements such as vacuum tubes and semiconductor devices. Apart from the sharp nonlinearities at the cut off and clipping points (see Fig 2), these devices also have nonlinear conduction mechanisms that give weakly nonlinear transfer functions also for bounded input signals.

The purpose of this chapter is to examine some of the properties of weakly nonlinear transfer functions and to give a frequency independent analysis of some basic amplifier structures. The results may also throw some light on the nonlinear behavior of the bipolar transistor, the behaviour of which can be regarded as the result of the composition of several weakly nonlinear transfer functions.

II:1 WEAKLY NONLINEAR TRANSFER FUNCTIONS

To try to distinguish between the different terms nonlinear, weakly nonlinear and linear, consider the graph of Fig.2. In general, this graph represents a nonlinear system. With proper restrictions on the input variable, we can define three operating regions. Region A, which includes the abrupt clipping point, has no limits on the input variable, and is called the nonlinear region. Region B, with input

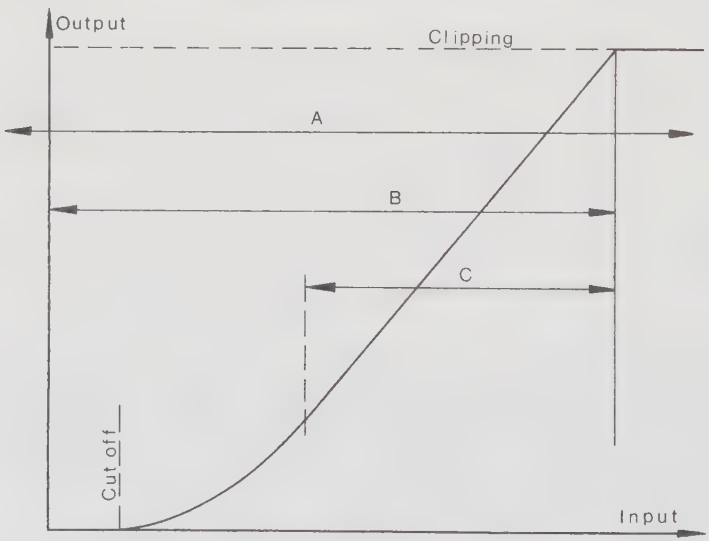

Fig.2. Schematic transfer function

signals above zero, including the smooth bend, but below the clipping point, we can call weakly nonlinear. Region C, the

straight line part, is called linear. Of course, this is not very useful, because it depends on how sharp the smooth bend is and on what we mean by abrupt.

A mathematical approach can be made by means of a series expression. Suppose that the transfer function of Fig.2 can be expressed as a power series of the input signal X:

$$Y = \sum_{k=0}^{N} b_k X^k \tag{1}$$

where it is assumed that terms above the Nth order have negligible importance.

Now let the quiescent operating point be given by X_o and Y_o. If equation (1) is then expressed as a Taylor series around the operating point [1], the terminal relationship becomes:

$$Y - Y_o = \sum_{k=0}^{N} a_k (X - X_o)^k \tag{2}$$

where the new coefficients a_k are given by:

$$a_k = \frac{1}{k!} \frac{d^k Y(X_o)}{dX^k}$$

To simplify the notation, let the incremental signals be denoted by:

$$y = Y - Y_o \quad \text{and} \quad x = X - X_o \tag{3}$$

Equation (2) then reduces to a power series of the incremental signal, i.e.:

$$y = \sum_{k=1}^{N} a_k x^k \tag{4}$$

Now we can make a quantitative definition of the degree of nonlinearity as the relative error made when approximating the series with the first linear term at a given input signal level:

$$\text{error} = \frac{\sum_{k=2}^{N} a_k x^k}{a_1 x} \tag{5}$$

In the following text we will assume a transfer function with the following property for bounded inputs.

$$a_1 x > \sum_{k=2}^{N} a_k x^k \tag{6}$$

This relation can also serve as a definition of weakly nonlinear transfer functions. In fact, this is not a very severe restriction, and most amplifying devices fall into this category.

The symbol used in the following text for an amplifying device in general is that of Fig.3, and it is assumed that the transfer function of this device will be weakly nonlinear.

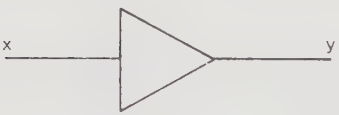

Fig.3. Amplifying device

II:2 HARMONIC GENERATION

When a sinusoidal signal is applied to the input of a nonlinear system, the output will contain the fundamental input frequency and multiples thereof. This follows directly from equation (4) when putting $x = x_p \sin\omega t$ and collecting terms with the same frequency, i.e.

$$y = A_1 \sin\omega t + A_2 \cos 2\omega t + A_3 \sin 3\omega t + \ldots \qquad (7)$$

The component with frequency $n\omega$ is called the n:th harmonic component, and the coefficient A_n is the amplitude of the n:th harmonic component.

As in section II:1, we can define a measure of the degree of nonlinearity using the amplitudes A_k:

$$\text{Thd} = \frac{\sqrt{\sum_2 A_k^2}}{|A_1|} \qquad (8)$$

This measure is called the total harmonic distortion or simply the distortion factor [1].

Depending on the application or the measurement technique, another definition, the n:th harmonic distortion, can be made [1]:

$$D_n = \frac{|A_n|}{|A_1|} \quad ; \quad n = 2,3,4\ldots \qquad (9)$$

And it is observed that the Thd can be expressed in the n:th harmonic distortion as:

$$\text{Thd} = \sqrt{\sum D_n^2} \qquad (10)$$

II:3 TRUNCATION ERRORS

There are two kinds of truncation errors made in this analysis. One from the Taylor-series approximation, and one from the reduced number of input sinusoidal amplitudes in each Taylor term.

Always when trying to make a finite power series match a measured transfer function, there will be an error due to the finite number of terms considered. Apart from model errors

there will also, for the same reason, be an error when expanding a theoretically deduced transfer function. The required number of terms depends, of course, on the application. As this text is concerned with weakly nonlinear functions, truncation after the first three terms is often adequate. In this case the error can be estimated with the fourth term.

The second kind of truncation error occurs in the amplitudes A_k in (7). These amplitudes contain powers of the input amplitude x_p. For example,

$$A_1 = a_1 x_p + 3/4\ a_3 x_p^3 + \ldots \qquad (11)$$

The error made when making the approximation $A_1 = a_1 x_p$ can be estimated as follows. Assume that a_k decreases at the rate $(k!)^{-1}$, and that $x_p = 1$. As will be seen in chapter VI:1 this is a worst case estimation (i.e. $D_2 = 25\%$). If the second term in the amplitude A_1 above is taken as a measure of the truncation error, the proposed condition will give an error of 13%. When making measurements to verify a given model this error can be controlled using a low enough input signal.

Thus, for weakly nonlinear transfer functions, and for moderate input amplitudes, it is justified to make the assumption:

$$A_1 = a_1 x_p \ ; \quad A_2 = 1/2 a_2 x_p^2 \ ; \quad A_3 = 1/4 a_3 x_p^3 \qquad (12)$$

An obvious conclusion of (11) is that distortion is strongly amplitude dependent.

II:4 WEAKLY NONLINEAR TRANSFER FUNCTION WITH FEEDBACK

Consider the amplifying device of Fig.3 but with feedback as in Fig.4, and suppose that the feedback network does not load the amplifier, i.e. the coefficients a_k remain the same after the introduction of feedback.

The effect on the transfer function when applying feedback is to produce an output y, which is a function of itself, i.e. an implicit function. Normally an implicit function is difficult

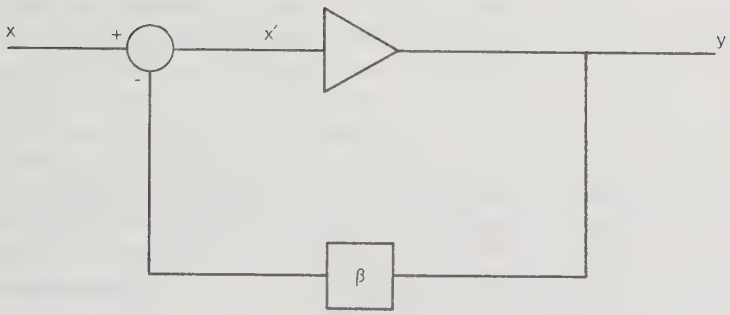

Fig.4 Weakly nonlinear transfer function with feedback

to solve, but when dealing with weakly nonlinear functions we can always define an inverse transfer function, i.e.

$$x = \sum b_k y^k \tag{13}$$

This inversion can be made either by means of direct series reversal as in appendix I or by implicit derivation as in appendix III. From Fig.4 we have $x' = x - \beta y$, and the inverse transfer function can be written:

$$x = (b_1 + \beta)y + b_2 y^2 + b_3 y^3 + \ldots \tag{14}$$

Taking the inverse of (14) defines the new transfer function with feedback:

$$y = \sum c_k x^k \tag{15}$$

The first three coefficients are found to be [2] :

$$c_1 = \frac{a_1}{(1 + \beta a_1)} \qquad c_2 = \frac{a_2}{(1 + \beta a_1)^3}$$

$$c_3 = \frac{a_3 - 2a_2^2\delta/a_1}{(1 + \beta a_1)^4} \qquad \delta = \frac{\beta a_1}{(1 + \beta a_1)}$$

It is obvious that feedback has a strong effect on distortion. For example the second order component is reduced by the factor $1/(1+\beta a_1)^3$, while the linear term is reduced by $1/(1+\beta a_1)$. An interesting fact is that the third and, in fact, all higher order coefficients can be nullified for specific sets of a_k and β. However, observe that nullification of, for example, the third order component demands the presence of a second order component. Another interesting fact is that the application of feedback to a transfer function of second order will produce a transfer function of higher order, i.e. produce harmonics of higher order than without feedback.

II:5 CASCADING WEAKLY NONLINEAR TRANSFER FUNCTIONS

Consider two noninteracting amplifying devices in cascade, as in Fig.5. The two systems are noninteracting in the sense that the transfer function of device number one remains unchanged after the cascade connection, as does that of device number two. The devices may also have local feedback. Suppose again that the two devices have transfer functions of the form (4), $x' = \sum a_k x^k$ and $y = \sum b_l x^l$ respectively.

Fig. 5 Cascaded weakly nonlinear transfer functions

Then the total transfer function is given by:

$$y = \sum b_1 \left(\sum a_k x^k \right)^1 = \sum c_n x^n \qquad (16)$$

Equating the three first coefficients gives:

$$c_1 = a_1 b_1 \qquad c_2 = a_1^2 b_2 + a_2 b_1$$

$$c_3 = a_1^3 b_3 + 2a_1 a_2 b_2 + a_3 b_1$$

A special case occurs when the second device has the inverse transfer function of that of the first, i.e.:

$$b_1 = \frac{1}{a_1} \quad ; \quad b_2 = - \frac{a_2}{a_1^3} \quad ; \quad b_3 = \frac{2a_2^2 - a_1 a_3}{a_1^5}$$

Under this condition we get the unity transfer function, i.e. $c_1 = 1$; $c_2 = 0$ and $c_3 = 0$. As discussed by Macdonald [3] it is possible to eliminate distortion components generated by a nonlinear device by applying complementary distortion before or after that device. The transfer function of the complementary system can be found by equating the coefficents c_k ; k = 2,3,4... of (16) to zero. Observe that using less than an infinite number of correction terms will produce a residual distortion.

A practical example of this technique is described, for example, in a paper of Bilotti [4].

II:6 BALANCED WEAKLY NONLINEAR TRANSFER FUNCTIONS

In the balanced amplifier structure, also known as the differential amplifier, the desired input signal is introduced differentially at the inputs. The desired output can be either differential or single ended. A balanced pair of transfer functions is outlined in Fig. 6.

The differential amplifier has many interesting features among which is the fact that it produces low second order distortion. The low distortion is automatically achieved when the output is taken as the difference between y_1 and y_2 . However, when a single ended output is desired, the amplifier of Fig.6 behaves just like a single unbalanced amplifier.

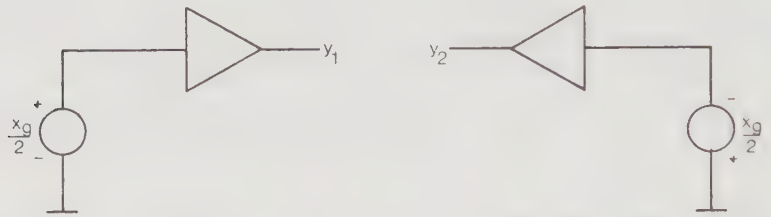

Fig.6 Balanced weakly nonlinear transfer functions

The performance of the single ended differential amplifier can be improved using common mode feedback. To demonstrate this, a more detailed amplifying device is needed.

Consider the configuration of Fig. 7, where two bipolar transistors are connected as a differential amplifier.

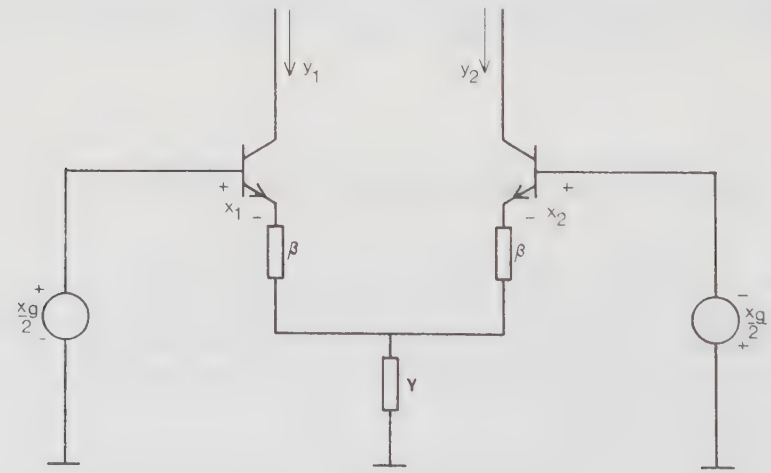

Fig.7. Bipolar transistor differential amplifier

Assume that the two transistors have identical transfer functions, and that the output currents can be expressed as power series of the input voltages, respectively. Assume also that the base current can be neglected in comparison with the collector current. Then the output currents are given by:

$$y_1 = \sum a_k x_1^k \qquad y_2 = \sum a_l x_2^l$$

The series feedback actions are performed by the impedances β and γ introduced in the input circuit. The β-feedback impedance is included to make a comparison with the single feedback amplifier possible. With reference to Fig. 7 the input equations are:

$$\begin{aligned} x_1 &= 1/2x_g - \beta y_1 - \gamma(y_1 + y_2) \\ x_2 &= -1/2x_g - \beta y_2 - \gamma(y_1 + y_2) \end{aligned} \tag{17}$$

The derivation of the resulting transfer functions follows the same method as for the single amplifier of chapter II:4, and the details are given in appendix II.

Thus, from appendix II and with the sign convention of Fig. 7, the output currents are given by:

$$\begin{aligned} y_1 &= d_1(x_g/2) + d_2(x_g/2)^2 + d_3(x_g/2)^3 + \ldots \\ y_2 &= -d_1(x_g/2) + d_2(x_g/2)^2 - d_3(x_g/2)^3 + \ldots \end{aligned} \tag{18}$$

where:

$$d_1 = \frac{a_1}{(1 + \beta a_1)} \qquad d_2 = \frac{a_2}{\lambda(1 + \beta a_1)^3} \qquad d_3 = \frac{a_3 - 2a_2^2\delta/a_1}{(1 + \beta a_1)^4}$$

$$\lambda = \frac{1 + \beta a_1 + 2\gamma a_1}{(1 + \beta a_1)} \qquad \delta = \frac{\beta a_1 + 2\gamma a_1}{(1 + \beta a_1 + 2\gamma a_1)}$$

The coefficients d_k appear to be similar to those of the single feedback amplifier.

The first coefficient, or the transfer gain d_1, is identical to the one of the single feedback amplifier.

The second term is changed by the factor λ in the denominator. This factor is known as the common mode rejection ratio, and is the ratio of the differential mode transfer gain to the common mode transfer gain. As can be observed in the equations (18), the signs of the second terms of the two functions are the same, which implies that these components appear in common mode and, consequently, each component experiences the additional feedback of the common mode impedance γ. If, for example, γ is the impedance of an active current generator, the common mode rejection ratio λ will tend to infinity, and the second order component will be suppressed to practically zero. It is interesting to notice that this suppression does not affect the transfer gain, and thus the differential amplifier is capable of selective feedback of the even order components.

The third component looks the same as for the single feedback stage, but there is a difference in the factor δ. This factor has the addition of the term $2\gamma a_1$ in both the numerator and the denominator.

In the single amplifier case the third order component could be nullified for specific sets of a_k and β. The question of a simultaneous suppression of the second term by maximizing γ, and nullification of the third term by proper adjustment of β, now arises. To investigate this, let us make an estimation of the expected magnitude of γ necessary for the nullification to occur.

As is normally the case for weakly nonlinear functions, the components a_k decrease with increasing k at the rate of $(k!)^{-1}$ i.e. $ca_1 = 1$, $ca_2 = 1/2$ and $ca_3 = 1/6$, where c is a constant. Thus the condition for nullification of the third component: $a_1 a_3 = 2\delta a_2^2$, will give $\delta = 1/3$ or explicily $\beta a_1 + 2\gamma a_1 = 1/2$. The maximum value of the common mode impedance is obtained for $\beta = 0$, and is $2\gamma a_1 = 1/2$. This estimation shows that the maximum value of λ for the third component cancellation to occur is $\lambda = 1.5$, and this is several orders of magnitude too small to suppress the second component significantly.

In the balanced output case, i.e. $y = y_1 - y_2$, it is observed from equation (18) that the second order components cancel regardless of the magnitudes of the feedback impedances β and γ.

Of course, any mismatch between the transfer functions of the two devices will violate this cancellation and produce a finite second order distortion.

III BIPOLAR TRANSISTOR MODELING

This chapter contains a description of two transistor models. The first one is the classic Ebers-Moll model, or to be precise, the transport formulation of this model [6] . It is based directly on semiconductor device physics and covers all operating regions. Only three model parameters need to be specified in order to fully describe it, namely, the reverse saturation current, and the forward and reverse current gains. The temperature is regarded as an operating condition. When restricted to the forward active region and a given quiescent collector current, the number of parameters reduces to one, namely, the forward current gain. This makes the model very simple and easy to use. Despite the model simplicity, it gives accurate estimations of input impedance and transconductance for bounded bias and operating conditions. However, when an extended range of operating and bias conditions are required, the EM model is insufficient.

It is even possible to make low frequency distortion predictions with this model, as will be shown in chaper VI:1.

The second model, the Integral Charge control Model presented by Gummel and Poon [7], takes into account many of the effects neglected by the EM model. Apart from high frequency effects the ICM also accounts for base width modulation (Early-effect) and variation of β with operating current. To fully define the ICM it is necessary to specify twenty-one model parameters. Fortunatly, when only the low frequency, forward active region is considered, the number of parameters required reduces significantly.

Also discussed in this section is the matter of obtaining model parameters.

III:1 EBERS-MOLL MODEL

The physical concept of the basic EM model is that of two diodes connected back to back and a current source in parallel with each diode. It is outlined in Fig.8 for a npn transistor [6] .

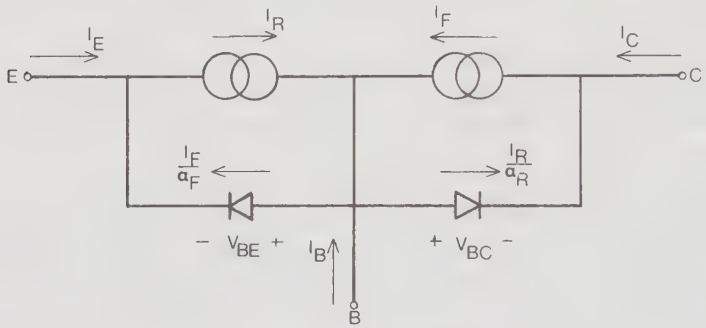

Fig 8 Transport version of Ebers-Moll model

The forward and reverse currents I_F and I_R represent those currents that are transported across the base and consist of diffusion currents. The forward and reverse reference currents can be written as [6] :

$$I_F = I_S(\exp(V_{BE}/V_T) - 1)$$
$$I_R = I_S(\exp(V_{BC}/V_T) - 1) \qquad (19)$$

As is seen above, the two currents have the same reverse saturation current I_S, which is one of the advantages of the transport formulation. The voltage V_T is the thermal voltage and is given by:

$$V_T = kT/q \tag{20}$$

where k is Bolzmanns constant, T is the absolute temperature and q the electron charge magnitude. The reverse saturation current is considered as constant, apart from a slight temperature dependence [5] .

With the reference polarity of Fig.8, the terminal currents are given by:

$$\begin{aligned} I_C &= I_F - I_R/\alpha_R \\ I_E &= - I_F/\alpha_F + I_R \\ I_B &= I_F(1/\alpha_F - 1) + I_R(1/\alpha_R - 1) \end{aligned} \tag{21}$$

Introducing the notation of common emitter current gain β, instead of the common base current gain α , the terminal currents become [6] :

$$\begin{aligned} I_C &= I_F - (1 + 1/\beta_R)I_R \\ I_E &= - I_F(1 + 1/\beta_F) + I_R \\ I_B &= I_F/\beta_F + I_R/\beta_R \end{aligned} \tag{22}$$

The corresponding total large signal equivalent circuit for the EM model can be drawn from this set of equations, as in Fig. 9. [6].

When operating in the forward active region, i.e. the base-emitter junction forward biased and the base-collector reversed biased, some simplification can be made as follows. For modern silicon transtors the value of I_S at room temperature is on the order of 10^{-15}A. The remaining model parameters β_F and β_R have typical values of 100 and 1 respectively. The thermal voltage is 26 mV at 300 K. Thus with the typical parameters above the contribution from the reverse collector current I_R and the reverse base collector current I_R/β_R can be neglected and the equations (22) can be reduced to:

$$I_C = I_S(\exp(V_{BE}/V_T) - 1)$$

$$I_E = -I_C(1 + 1/\beta_F) \qquad (23)$$

$$I_B = I_C/\beta_F$$

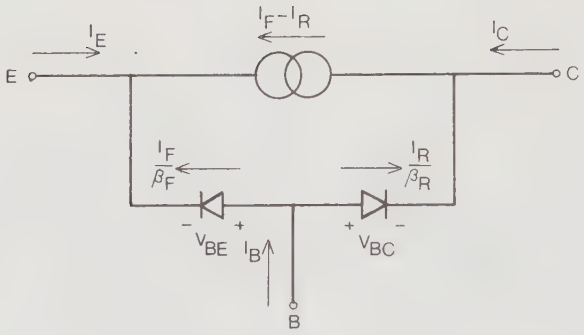

Fig.9 Ebers-Model large signal equivalent circuit

The corresponding forward large signal equivalent circuit for a common emitter configuration is depicted in Fig.10 [6].

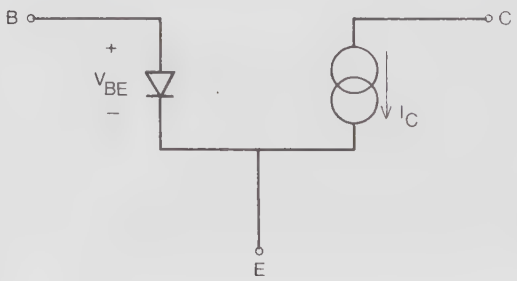

Fig 10. Forward region nonlinear equivalent circuit

III:2 LINEARIZED EM-MODEL

A small signal version of the EM model can easily be derived for the low frequency forward active region using equation (23). Let the quiescent collector current be denoted I_o and the corresponding base emitter voltage V_o. Then from equation (23) we have:

$$I_o = I_S(\exp(V_o/V_T) - 1) \tag{24}$$

Letting i_c be the incremental collector current as a response to an incremental bas-emitter voltage v_{be}, the Taylor series expansion of the total collector current is given by:

$$I_o + i_c = I_o + \frac{I_o}{V_T} v_{be} + \frac{I_o}{2V_T^2} v_{be}^2 + \frac{I_o}{6V_T^3} v_{be}^3 + \ldots \tag{25}$$

The small signal equivalent circuit for a common emitter configuration at operating current can be drawn directly from equation (25), as in Fig.11, where the transconductance g_m is given by $g_m = I_o/V_T$.

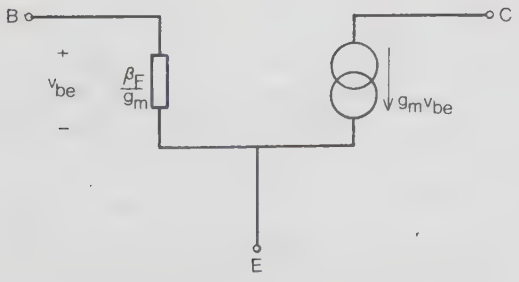

Fig.11 Linearized forward Ebers-Moll equivalent circuit

Since in this first order model, β_F is a constant, the incremental base current is given by $i_b = i_c/\beta_F$, and the input base-emitter impedance is β_F/g_m.

When comparing data book values for the common emitter h-parameter h_{ie} with the input impedance of this derivation, the agreement is very good for low currents, but for higher currents the above model fails. This in mainly due to two effects omitted in this EM model: parasitic series resistances and high injection. These effects will be included in the Gummel-Poon model described in section III:5.

III:3 EBERS-MOLL PARAMETERS

The three parameters I_S, β_F and β_R of the EM-model can be extracted from a simultaneous measurement of I_C and I_B as functions of V_{BE} in both the forward and inverse regions. In the forward region, the base-collector junction should have zero or constant reverse bias, and the emitter-base should have zero or constant reverse bias in the inverse region . A result of such a measurement in the forward region, for the modern small signal silicon transistor BC 550C, is given in Fig. 12. As is seen in this diagram, the collector current is a straight line for low current as predicted by the EM-model. The extrapolated curve to zero base-emitter voltage gives the reverse saturation current I_S. The slope of this curve should be equal to the thermal voltage V_T^{-1}.

The EM-model predicts that the base current curve should be parallel to the collector current curve in this diagram. This, however,is not the case. The slope of the base current tends to vary as base-emitter voltage is increased from $(2V_T)^{-1}$ at low values, to V_T^{-1} at higher. This is, however, in agreement with experience and the conclusion is that the EM-model fails to explain this variation.

This implies that β_F is a function of base-emitter voltage and hence of collector current. Consequently when this parameter is important, either a mean value over the current range of interest has to be computed, or the specific value of the operating point should be used.

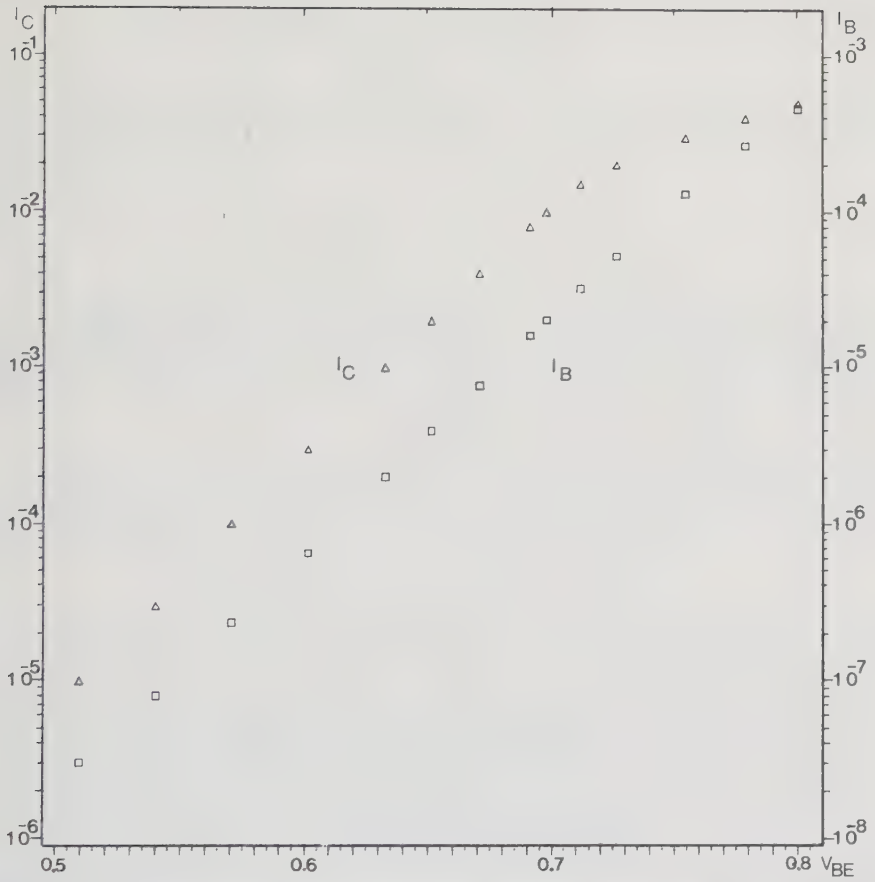

Fig.12 Collector and base currents versus base-emitter voltage

III:4 INTEGRAL CHARGE CONTROL MODEL

The Integral charge control model (ICM) presented by Gummel and Poon in 1970 [7] is based on the transport version of the EM model, with some of its idealisations removed. Many of the additional effects accounted for by the ICM are treated in a unified manner through a modulation of the forward and reverse reference currents. The modulator is the normalized base charge

q_B, which is the ratio of the total majority base charge Q_B at the bias point and its value at zero bias Q_{BO}, i.e:

$$q_B = Q_B/Q_{BO} \tag{26}$$

Then the new forward and the reverse currents are [8] :

$$\begin{aligned} I_F &= I_S/q_B(\exp(V_{BE}/V_T) - 1) \\ I_R &= I_S/q_B(\exp(V_{BC}/V_T) - 1) \end{aligned} \tag{27}$$

The total majority base charge Q_B consists of capacitively stored charges originating from the base-emitter and the base collector space charge regions, and of products of collector current components and transit times, i.e diffusion charges.

Normalization with respect to the zero bias value gives the following expression [6]

$$q_B = 1 + q_{jE} + q_{jC} + q_{dE} + q_{dC} \tag{28}$$

where: q_{jE} is the normalized majority charge related to the base-emitter junction capacitance C_{jE}

q_{jC} is the normalized charge contribution from the base-collector junction capacitance C_{jC}

q_{dE} is the normalized charge of the base-emitter diffusion capacitance C_{dE}

q_{dC} is the normalized charge of the base-collector diffusion capacitance C_{dC}

At this point, it is convenient to make some approximations relevant to the forward active region of operation. First of all, the reverse current I_R can be neglected because of the reverse bias on the base-collector junction. Furthermore, the base-emitter depletion capacitance, and hence the charge component originating from this capacitance, is of little importance for low frequencies. The low frequency effect of this charge component is to produce a collector current slope that deviates from V_T^{-1}. This deviation is often expressed as an emission coefficient n_e, and the resulting collector current slope is $(n_e V_T)^{-1}$ [5]. Thus, with these simplifications, the normalized base charge in the forward active region is,[11]:

$$q_B = 1 + q_{jC} + q_{dE} \tag{29}$$

The reason for not neglecting the base collector charge contribution with the low frequency argument is that this term models the collector conductance, and this effect is indeed a low frequency effect.

The remaining expression of q_B can be given a more measurable form by the introduction of two model parameters, the Early voltage V_A and the knee curent I_K. The parameter V_A models the output conductance, and the parameter I_K models the onset of high injection. Thus the normalized base-collector charge is written:

$$q_{jC} = -1/V_A \int_0^{V_{CE}} C_C(V)/C_0 dV \tag{30}$$

Neglecting the base pushout factor the normalized base-emitter charge component is then expressed as [6]:

$$q_{DE} = I_F/I_K \tag{31}$$

It is often relevant to write $I_F = I_C$ in this expression. The experimentally observed deviation of the base current from a straight line when plotted in a semilog diagram as a function of base-emitter voltage is also accounted for in the ICM. This is done through an additional nonideal base current component which models the extra recombination currents identified in the ICM. Thus for the forward region the base current can be modeled as [9] :

$$I_B = I_S/\beta_{FM}\exp(V_{BE}/V_T) + CI_S\exp(V_{BE}/nV_T) \tag{32}$$

where C, β_{FM} and n are aditional model parameters.

To summarize the ICM for the low frequency forward active operating region, the following set of equations will suffice [9] :

$$\begin{aligned} I_C &= I_S/q_B\exp(V_{BE}/V_T) \\ I_B &= I_S/\beta_{FM}\exp(V_{BE}/V_T) + CI_S(\exp(V_{BE}/nV_T) \\ q_B &= 1 + q_{jC} + q_{dE} \end{aligned} \tag{33}$$

Thus the additional effects accounted for by the low frequency ICM are high injection and thereby high current falloff of current gain β_F. This effect in modeled with the parameter I_k. The collector current dependence on the collector emitter voltage is modeled by the parameter V_A. The current gain fall off at low current levels is described through a proper modeling of the base current whith the parameters β_{FM}, C and n.

III:5 PARASITIC SERIES RESISTANCES

As can be observed in Fig. 12, the collector and base currents versus base-emitter voltage curves suffer from nonideal behavior at high current levels. This effect is partly caused by the voltage drop across the ohmic base and emitter series resistances, denoted r_{bb} and r_{ee}, respectively.

The collector series resistance r_{cc} has its greatest influence in the saturation region and has negligible importance in the forward active region.

The ohmic base resistance r_{bb} is composed of two parts: one constant part originating from the base terminal, and one current dependant part from the inactive base region [22].

The ohmic emitter resistance r_{ee} is also composed of one constant and one current dependent component, originating from the emitter terminal and the inactive bulk emitter region, respectively. [22] .

However, the current dependent part is for both resistances small compared with the constant part. Thus the parasitic resistances will be considered constant. The magnitude of r_{bb} is on the order of 10Ω , and r_{ee} is on the order of 0.1Ω .

If we assume that the base and emitter resistances are connected as in Fig.13, the terminal V_{BE} and intrinsic V'_{BE} base-emitter voltages will be related as:

$$V'_{BE} = V_{BE} - (r_{bb} + r_{ee})I_B - r_{ee}I_C \qquad (34)$$

For simplicity we can define a model equivelent series resistance r_{eq} as:

$$r_{eq} = (r_{bb} + r_{ee})1/\beta_{DC} + r_{ee} \qquad (35)$$

When making DC measurements, the voltage drop across this resistance should be considered. For low frequency AC measurements the parasitic resistance can normaly be neglected.

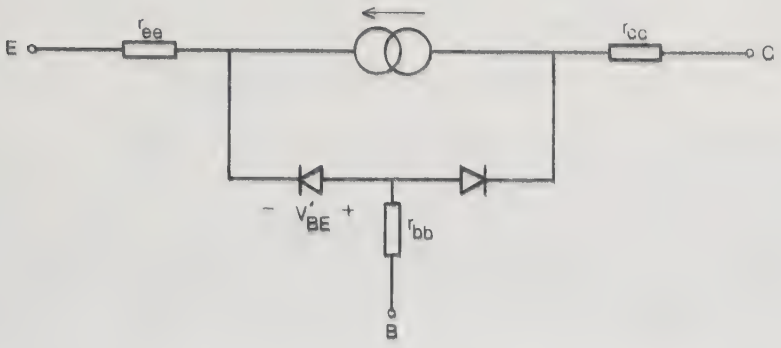

Fig.13 Parasitic series resistances connection

III:6 ICM PARAMETERS

The complete ICM demands the specification of twenty-one parameters, but with the reduction to low frequencies and forward active region of operation there are only six left. These parameters as they appear in (33) are:

I_S the reverse saturation current

I_K the knee current, (defining the onset of high injection)

V_A the Early voltage (output conductance parameter)

β_{FM} the maximum forward current gain

C nonideal base current coefficient

n nonideal base current emission coefficient.

To illustrate the definition of the above parameters, consider again a semilogarithmic plot of the collector and base currents versus the base emitter voltage for constant base collector voltage as is outlined for a fictive transistor in Fig. 14. At low forward bias well below the knee point I_K, the collector current has an ideal slope of V_T^{-1}, i.e.:

$$I_C << I_K \qquad I_C = I_S \exp(V_{BE}/V_T) \tag{36}$$

By extrapolating this region to zero base emitter voltage, we obtain the value of I_S.

For higher bias levels the slope of the collector current tends to level off, and for high injection, i.e. collector current well above the knee point, the slope becomes nonideal and approaches $(2V_T)^{-1}$.

$$I_C >> I_K \qquad I_C = \frac{I_S \exp(V_{BE}/V_T)}{I_C/I_K} = \sqrt{I_K I_S}\exp(V_{BE}/2V_T) \tag{37}$$

If it were possible to assume that these two regions were well separated, a value of the knee current I_K could be obtained from the intersection of the two asymptotes. However, for most transistors the two regions overlap. Also, the voltage drop across the bulk base and emitter resistances will cause a slope which is less then $(2V_T)^{-1}$.

The base current also has two asymptotes. Starting in the low current region the second term of the base current dominates giving a non-ideal slope of $(nV_T)^{-1}$:

$$\text{low current asymptote} \qquad I_B = CI_S \exp(V_{BE}/nV_T) \tag{38}$$

Again by extrapolating this region to zero V_{BE} makes it possible to determine the parameter C, and the slope of this asymptote determines the parameter n.

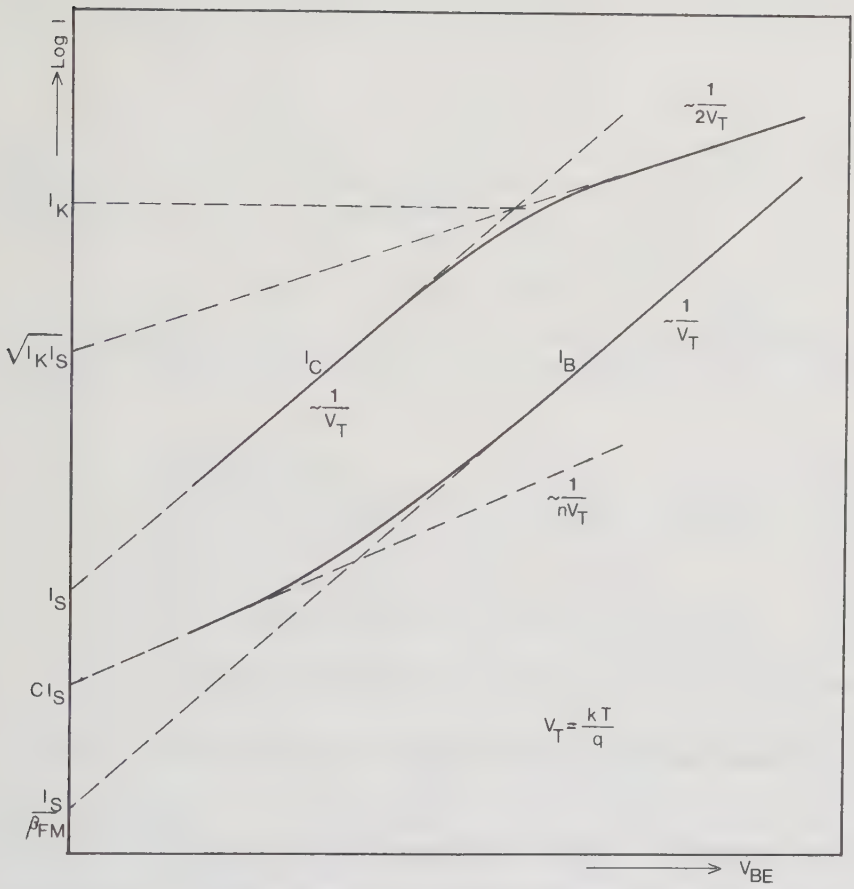

Fig.14 Illustration of ICM parameter definition

At medium bias levels the ideal base current component dominates and the slope is V_T^{-1} :

$$\text{medium asymptote} \qquad I_B = I_S/\beta_{FM}\exp(V_{BE}/V_T) \qquad (39)$$

At high current levels the voltage drop across the bulk base and emitter resistances becoms significant, making the base current slope less than the ideal V_T^{-1}.

Combining the two curves in Fig.14 to form the current gain versus collector current I_C (as in the diagram of Fig. 15), gives another wiew of the parameters β_{FM}, n, C and I_K. Starting again in the low current region we have, from (36) and (38):

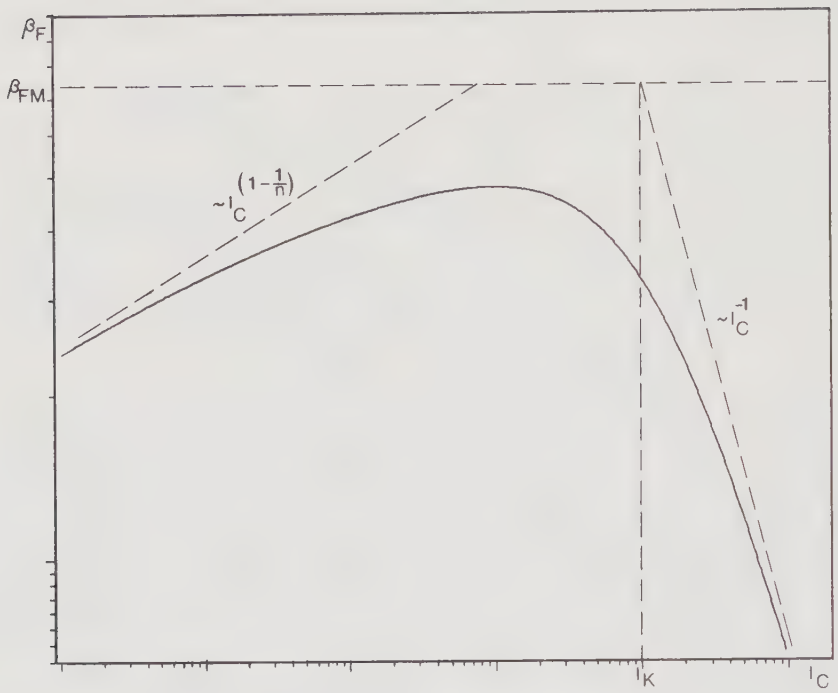

Fig.15 Common emitter current gain versus collector current

$$\text{low currents} \quad \beta_F = I_C/I_B = C^{-1} I_S^{(1/n-1)} I_C^{(1-1/n)} \qquad (40)$$

If we take the logarithm of (40), we see that the low current slope is (1-1/n).

Proceeding to medium currents but still below I_K, it is seen from (36) and (39) that β_F is constant with the value β_{FM}.

Above the knee current I_K, β_F will decrease at a rate given by (37) and (39) i.e.:

$$\text{high currents} \qquad \beta_F = \beta_{FM}\ I_K\ I_C^{-1} \qquad (41)$$

When dealing with real transistors these idealized regions overlap and no clear boundaries are visible between them, causing difficulties in the parameter determinations. For example, the maximum measured forward current gain is often less than the model parameter β_{FM}.

For the low injection region one can make a simple interpretation of the output conductance parameter V_A: Expression (29) reduces to

$$q_B = 1 + q_{jC} \tag{42}$$

Since V_A is normally very large (~ 100V) this makes q_{jC} small compared to unity, and the following approximation for the collector current is appropriate [6] :

$$I_C = (1 - q_{jC}) I_S \exp(V_{BE}/V_T) \tag{43}$$

Evaluating the integral of (30) with the mormal expression [6] for the junction capacitance, and assuming that $V_{BC} = - V_{CE}$ gives:

$$q_{jC} = - \frac{1}{V_A} \int_0^{V_{CE}} \frac{dV}{(1 + V_{CE}/\varphi)^m} = - \frac{1}{V_{A(1-m)}} [(1 + V_{CE}/\varphi)^{(1-m)} - 1] \tag{44}$$

where: φ is the built in voltage (~ 0.7V)
m is the gradient factor (0.33 < m < 0.5).

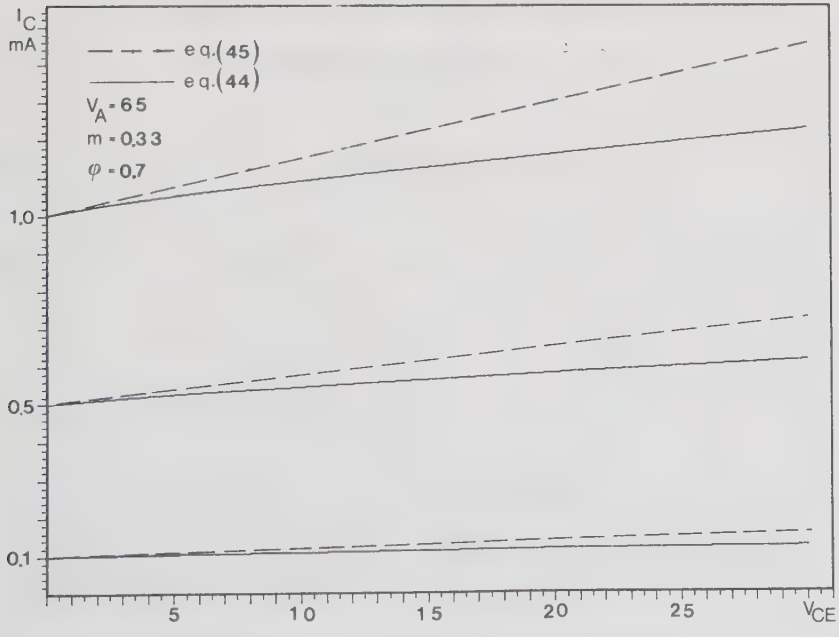

Fig.16 Collector current versus collector emitter voltage

For low collector emitter voltage this expression can be approximated with [6].

$$q_{jC} = - V_{CE}/V_A \qquad (45)$$

The low voltage approximation of q_{jC}, together with the full expression of equation (44), is outlined in Fig. 16.

III:7 LINEARIZED ICM

A linearized version of the low frequency ICM can be derived using equation (33).

Consider the equivalent circuit of Fig. 17, where the components are defined as:

$g_i = dI_B/dV_{BE}$ the input conductance

$g_m = dI_C/dV_{BE}$ the transfer conductance

$g_o = dI_C/dV_{CE}$ the output conductance

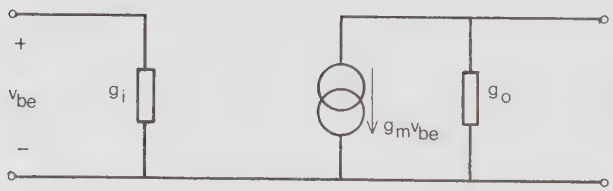

Fig.17 Linearized low frequency ICM

It is observed that the parameter g_i is normally paralleled with a current source representing feedback from the output terminal. However, since the ICM does not account for reverse feedback [36], this current source has been omitted.

The derivation of the input conductance from the I_B expression of (33) is straightforward, i.e:

$$g_i = I_S/(V_T\beta_{FM})\exp(V_{BE}/V_T) + CI_S/(nV_T)\exp(1/nV_{BE}/V_T) \quad (46)$$

It is convenient to express the small signal parameters in terms of collector current. Substituting the collector current from (33) into the above one we have:

$$g_i = I_C q_B/(V_T\beta_{FM}) + C/(nV_T)I_S^{(1-1/n)}I_C^{1/n} \quad (47)$$

The transconductance g_m and output conductance g_o are derived using an implicit method, as in appendix III. The expressions for g_m and g_o are:

$$g_m = \frac{I_C}{V_T[1 + I_C/(I_K q_B)]} \quad (48)$$

$$g_o = \frac{I_C}{V_A(q_B + I_C/I_K)} \quad (49)$$

Another small signal parameter of interest is the incremental current gain B_{ac}, defined as [5] :

$$\beta_{ac} = \{[1 + I_C/(I_K q_B)][q_B/\beta_{FM} + C/nI_S^{(1-1/n)} I_C^{(1/n-1)}]\}^{-1} \quad (50)$$

IV DISTORTION EXPRESSION

As described in section II, the distortion expression is found by expanding the transfer function into a Taylor series or, in other words, by computing the derivatives with respect to the driving variable.

Derivation of the full description of the ICM gives rise to large algebraic expressions which are difficult to handle. Therefore, some modifications in the DC equations are made to facilitate the derivation procedure.

In this chapter we will include all DC effects, i.e. β_{DC} variation with current and the Early-effect, but with a slight modification in the description of these effects. The modification lies in the formulation of the normalized base charge q_B, which, in this case, is considered close to unity, i.e. the collector emitter voltage is less than the Early voltage and the collector current is less than the knee current. From (33) one can write this modification:

$$I_C = \frac{I_S \exp(V_{BE}/V_T)}{(1 + q_{jC} + I_C/I_K)} \approx I_S \exp(V_{BE}/V_T)(1 - q_{jC} - I_C/I_K) \qquad (51)$$

The departure from the original ICM is outlined in Figure 18. It is evident from this figure that this description of high injection for currents near the knee current is not close to the ICM, although this is not as severe as it appears. For low power devices the power dissipation sets the upper limit of the operating current to well below the knee current. For example, the BC 550C has a knee current of approximatly 50 mA and a maximum power dissipation capability of 0.5W. Thus, for a collector-emitter voltage of 15 volts, the current is limited to 33 mA.

The modified description of the Early effect is very close to the ICM, and together with the low voltage approximation of (45) this leeds to the often used description of the Early-effect for low currents [9][10]

$$I_C = I_S \exp(V_{BE}/V_T)(1 + V_{CE}/V_A) \qquad (52)$$

The derivation will be performed under the assumption that there are resistive voltage drops in the base, emitter and collector leads as depicted in Fig.19. These resistive elements do not need to be ohmic, but they are assumed to be linear.

The derivating procedure is given in appendix III and only the resulting second order distortion expression is given in section IV:1.

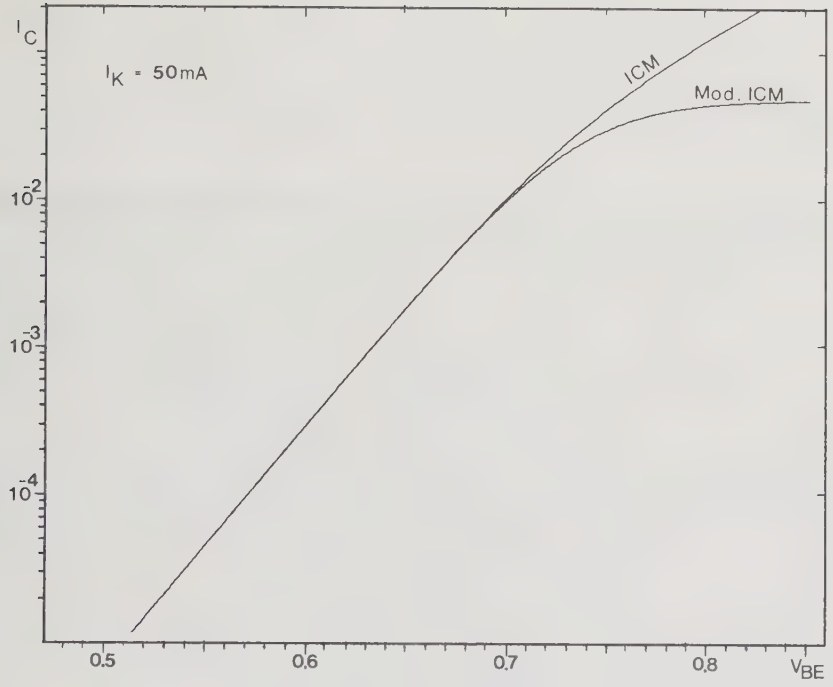

Fig.18 Modified ICM collector current versus V_{BE}

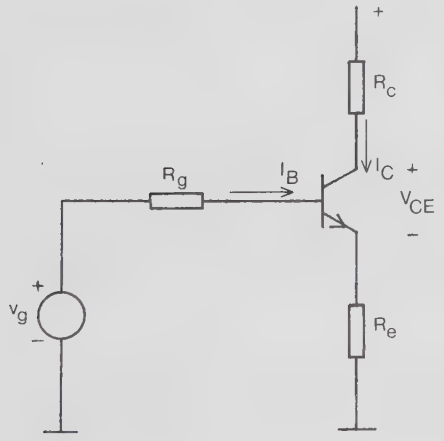

Fig.19 Amplifier for which the distortion is derived

IV:1 SECOND ORDER DISTORTION

From appendix III we have the second order harmonic distortion for a common emitter stage as:

$$D_2 = \frac{\{1 - [I_C q'/(1-q)]^2 - I_C{}^2 R'_{eq}/V_T\}}{4V_T[1 + I_C q'/(1-q) + I_C R_{eq}/V_T]^2} v_p \qquad (53)$$

where:

$$q = - V_{CE}/V_A + I_C/I_K$$

$$q' = (R_c + R_e)/V_A + 1/I_K$$

$$R_{eq} = (R_b + R_e)dI_B/dI_C + R_e$$

$$R'_{eq} = (R_b + R_e)d^2I_B/dI_C{}^2$$

It can be seen that the second order distortion can be nullified by proper adjustment of q' and R'_{eq} for a given collector current.[22],[24],[25]. The magnitudes of q' and R'_{eq} can be controlled by means of the external components in the output and the input loops.

The resistive components in the output loop, i.e. the collector and emitter resistances, act as feedback elements from the collector current to the collector-emitter voltage. In a similar manner, the base and emitter resistances act as feedback elements from the base and collector currents to the base-emitter voltage.

To get qualitative information on how these external components affect the second order distortion, we will study the distortion as a function of the external componenents separately. But first the effects of the external components on q' and R'_{eq} will be investigated.

IV:1:1 EARLY EFFECT INFLUENCE ON SECOND ORDER DISTORTION

At a given quiescent current the Early effect influences the second order distortion through the term containing the factor q' in equation (53). For the low injection case this factor can be given its simplest form with the low collector emitter voltage approximation as :

$$q' = dq/dI_C = R_C/V_A \tag{54}$$

Using the small signal output conductance of equation (49), together with the modified ICM description and assuming zero base emitter equivalent resistance, equation (53) can be rewritten in a simpler form, as below:

$$D_2 = \frac{(1 - R_C g_o)}{4V_T(1 + R_C g_o)} v_p \tag{55}$$

where: $g_o = I_C/(V_A + V_{CE})$

This equation describes the combined effect of the base-emitter and the output nonlinearities.

For zero collector resistance it reduces to the result obtained from the Ebers-Moll equations, as could be expected. Increasing the collector resistance to the value of the output impedance of the transistor at the operating current, the output nonlinearity will cancel the second order base-emitter nonlinearity. Further increase leads to a phase change in the total second order distortion and its magnitude again approaches the Ebers-Moll result.

The Early effect also influences the second order distortion via the current gain, which will be discussed in the next section.

IV:1:2 EQUIVALENT SERIES RESISTANCE INFLUENCE ON DISTORTION

The equivalent series resistance (R_{eq}) can be viewed as a current series feedback from the collector current to the base-emitter voltage. Because of the relationship between the base and collector currents, this resistance also provides feedback from the base current to the base emitter voltage. Apart from the linearizing effect of the term containing R_{eq} in the denominator of equation (53), the numerator includes the derivative of R_{eq}. This derivative is current dependant and changes sign from negative to positive as the current is increased. The current dependency arises from the second derivative of the base current.

From equation (33) and with the notation of (AIII:2), we can express the base current in terms of the collector current. Using a simple interpretation of the base current suggested by Unger and Berbalk [11] we can write:

$$I_B = I_C/[\beta_{FM}(1 - q)] + CI_S^{(1-1/n)}I_C^{1/n} \qquad (56)$$

The approximation lies in the second low current term in which the factor (1-q) has been omitted. Deriving this expression with respect to the collector current gives:

$$dI_B/dI_C = 1/[\beta_{FM}(1-q)][1 + I_C q'/(1-q)] + \\ + C/nI_S^{(1-1/n)} I_C^{(1/n-1)} \qquad (57)$$

$$d^2I_B/dI^2_C = \frac{2q'}{\beta_{FM}(1-q)^2}[1 + I_C q'/(1-q)] + \\ + C/n(1/n-1)I_S^{(1-1/n)}I_C^{(1/n-2)} \qquad (58)$$

The second order base current derivative is plotted in Fig. 20 as a function of collector current for different collector load impedances.

As is indicated in this figure, the second derivative is negative for low currents, which for example implies that increasing the base-resistance R_b in thisregion of operation

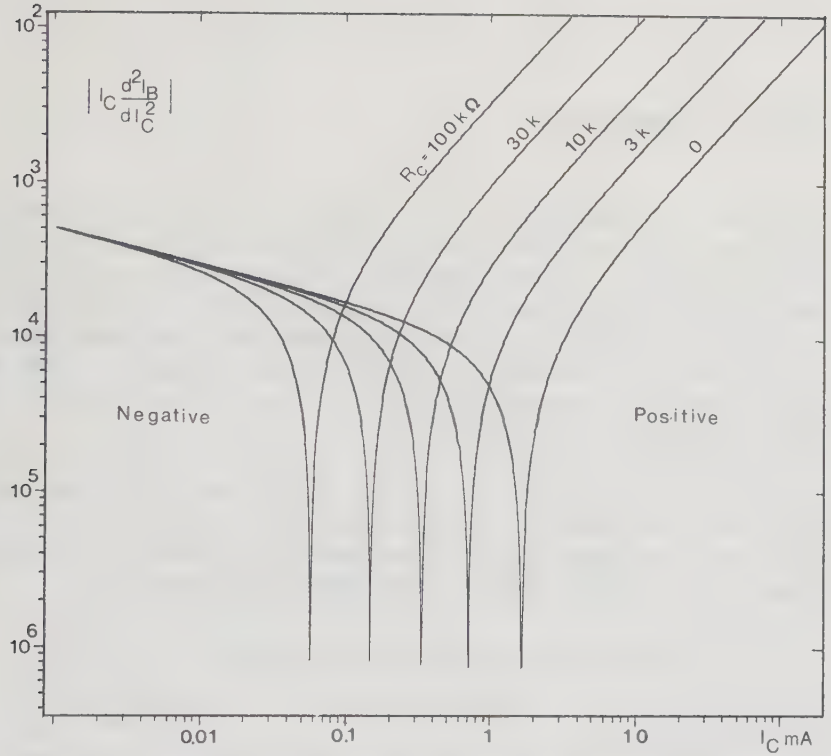

Fig.20 Second order base current derivative versus I_C

the numerator of equation (53) will counteract the linearizing effect of the denominator. However, the magnitude of the second derivative is usually very small, which indicates that the linearizing effect will dominate. For currents approaching the knee current, the R_{eq} and R'_{eq} terms cooperate and the sign of R'_{eq} makes cancellation of second order distortion possible [24]. The Early effect also has an interesting influence on the equivalent series resistance. Increasing the collector resistance will result in a decrease in the zero point of the second base current derivative. This means that the point where the R_{eq}' term changes sign can be controlled by means of the collector resistance.

The emitter resistance R_e has its main influence in the denominator of equation (53). The combined effect of the collector and emitter resistances is that R_e has a strong linearizing effect below the collector resistance cancellation point, but for R_c above this point the effect of R_e will diminish.

V PARAMETER MEASUREMENTS

As discussed in chapter III:6 there is a great deal of information that can be extracted from a simultaneous measurement of the collector and base currents as functions of base-emitter voltage. The only problem when making manual measurements is to keep the collector-base voltage constant or zero. Care must also be taken in order not to enter into the saturation region because of the voltage drop across the collector parasitic resistance.

Measurements have been made on the low power transister BC 550C.The obtained data are presented in Figs. 21 to 22. By adjusting the parameters of the modified model equation, theoretical curves were made to fit these data points.

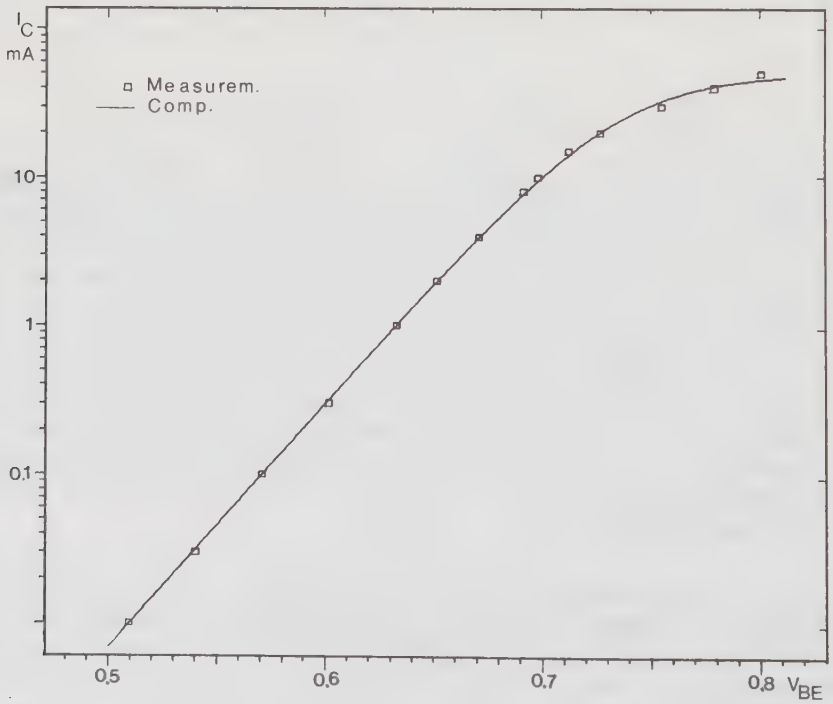

Fig.21 Collector current versus V_{BE}

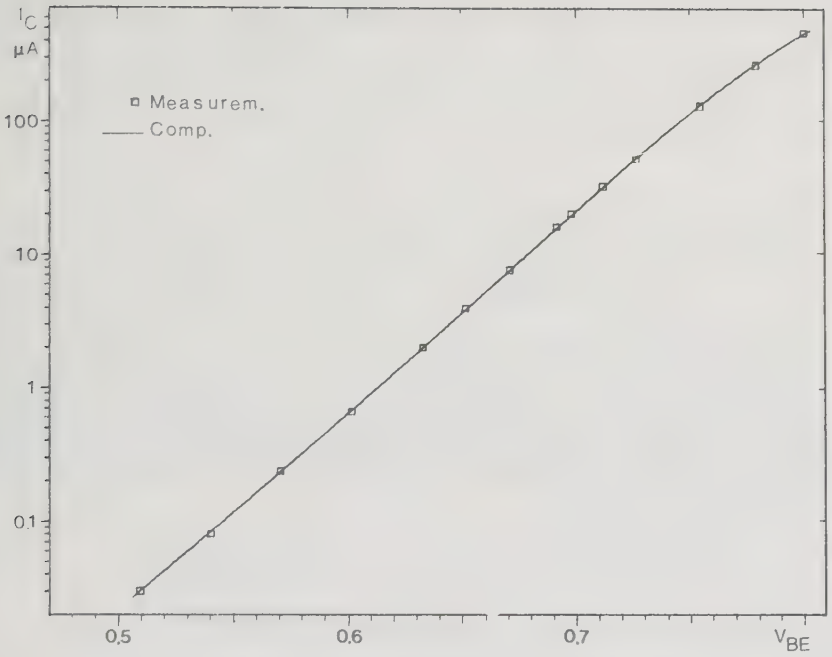

Fig.22 Base current versus V_{BE}

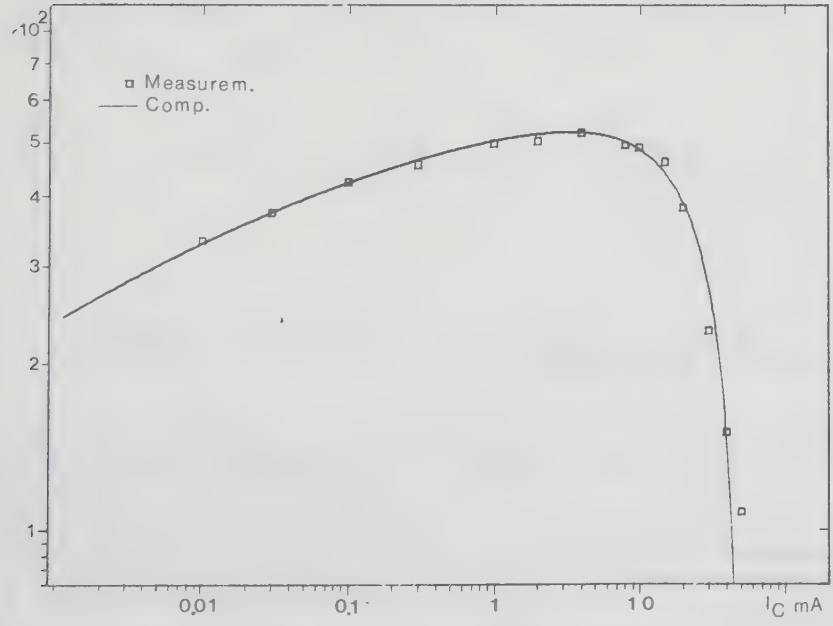

Fig.23 Comon emitter current gain versus I_C

Because of the resistive voltage drop across the parasitic resistances at high current levels it is difficult to estimate the high collector current asymptote from the I_C - V_{BE} diagram. A better estimation is obtained from the β_{DC} - I_C diagram as in done in Fig. 23 above.

The output conductance parameter V_A can be obtained by measuring the collector current for constant base-emitter voltage for various collector emitter voltages. The power dissipation must be kept to a minimum in this type of measurement in order to avoid temperature effects. An alternative way of obtaining the parameter V_A at higher bias levels is to measure the collector current change as a response to a collector emitter voltage change. The result of such a measurement in the range 0.1 to 8mA and for V_{CE} = 15V is given in the summary below. Observe that V_A is a constant only for currents below 1mA [19].

Summary of the obtained parameters for the BC 550C:

$I_S = 4.6 \cdot 10^{-14}$ A ; I_K = 50mA ; β_{FM} = 730

C = 0.14 ; n = 1.3 ; r_{eq} = 0.05Ω

I_C	0.1	0.5	1.0	2.0	4.0	8.0	mA
g_o	1.3	6.3	12	27	63	143	μA/V
V_A	65	65	65	57	47	35	V

VI DISTORTION MEASUREMENTS

This chapter will focus on harmonic distortion measurements of two basic amplifiers, the differential pair and the common emitter stage. The aim of these measurements has been to obtain information concerning the mechanisms governing distortion generation and the influence of the external components on this generation.

The first series of measurements described was performed to confirm the Ebers-Moll model in a common emitter stage with series current feedback.

The second measurement series was made on a balanced differential amplifier and is also pertinent to the Ebers-Moll model and to the results of chapter II:6.

The third series of measurements was made in order to apply the ICM to a common emitter stage and was therefore focused on the effect of high generator and load impedances.

The technique chosen for the distortion measurement was that of feeding the test amplifier with a pure sine wave signal and analysing the output signal with a tunable narrow band analyser. This technique has been chosen because of its high noise rejection. The measurement setup is outlined in Fig. 24.

Fig.24 Distortion measurement setup

VI:1 MEASUREMENTS PERTAINING TO THE EBERS-MOLL MODEL

For the restrictions of low currents and low generator and load impedances the ICM reduces to the Ebers-Moll model.

To meet the above restrictions, a cascade connection as shown in Fig.25 can be employed, either with an operational amplifier or with a darlington transistor. Beginning with zero emitter resistance the harmonics were measured for various generator voltages. The result is given in the diagram of Fig. 26.

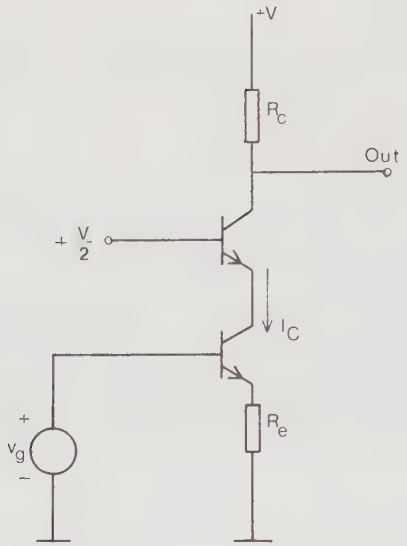

Fig.25 EM model test amplifier

As expected, the second order distortion is proportional to the generator voltage and the third order is proportional to the square of this voltage, which indicates that the truncation (12) in chapter II:3 is justified in this case.

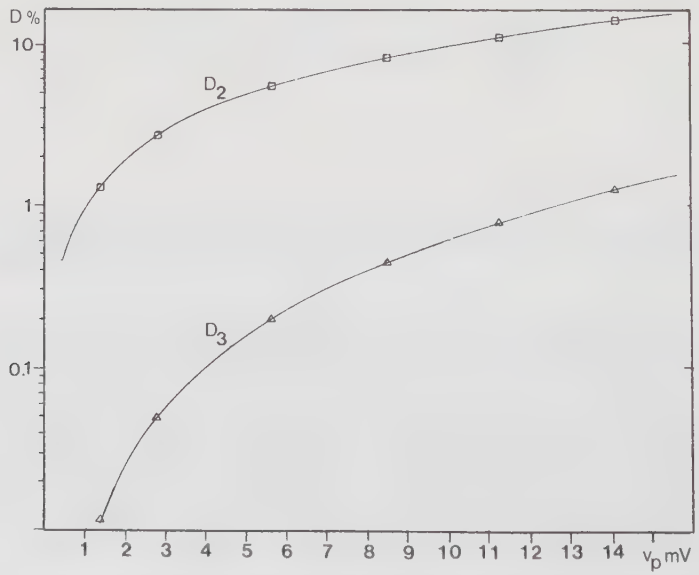

Fig26 Second and third order distortions as functions of generator peak voltage v_p

Applying feedback by the resistance R_e will decrease the distortion as shown in Fig. 28 for the second and third order harmonics. From equations (15) and (25) we have, for this case [18] :

$$D_2 = \frac{1}{4V_T(1 + R_e g_m)^2} v_p$$

$$D_3 = \frac{(1 - 2R_e g_m)}{24V_T^2(1 + R_e g_m)^4} v_p^2 \qquad (59)$$

where $g_m = I_C/V_T$, and v_p is the peak generator voltage.

As expected the third order distortion has a sharp minimum for the feedback resistance $R_e = g_m/2$. The results presented in Fig.27 were obtained by using an emitter resistor of 10Ω and adjusting the quiescent current from 0.15 to 10 mA.

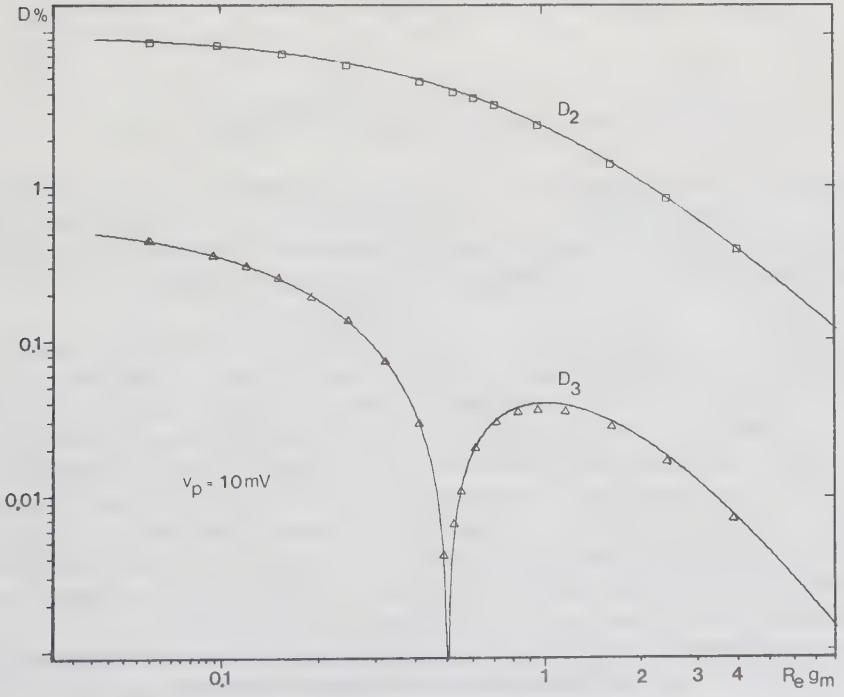

Fig.27 Second and third order distortions as functions of normalized feedback $R_e g_m$

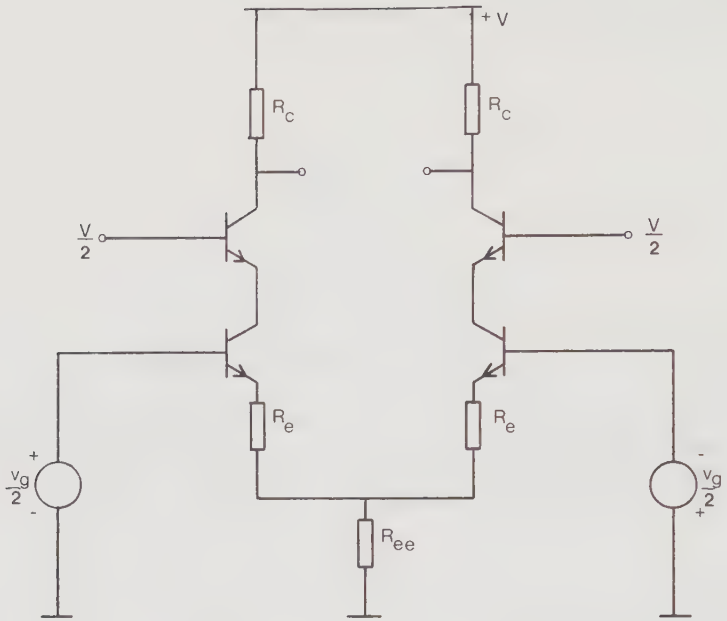

Fig.28 EM model differential test amplifier

Measurements have also been made on a differential amplifier in order to investigate the interaction between the two transistors via the common mode impedance. The test amplifier is outlined in Fig. 28 without its biasing components.

As mentioned in chapter II:6 the two transistors should have as equal characteristics as possible. Therefore, a matched pair was choosen as a test object. Furthermore, the collector currents were balanced by means of the external circuitry before the measurement procedure. Each side of the differential amplifier was analysed separately. The three first output harmonics for constant input were analysed for collector currents in the range 0 .1 to 8 mA, and for common emitter impedances from zero up to ten kiloohms. The measured distortions are shown in Figs. 29 and 30, as functions of $R_e g_m$ together with theoretically deduced data. From the equations (18) and (25), the second and third order distortions are given as follows:

$$D_2 = \frac{1}{4V_T\lambda(1 + R_e g_m)^2}(v_p/2)$$

$$D_3 = \frac{(1 - 2(R_e + 2R_{ee})g_m)}{24V_T^2\lambda(1 + R_e g_m)^4}(v_p/2)^2 \qquad (60)$$

$$\lambda = \frac{(1 + (R_e + 2R_{ee})g_m)}{(1 + R_e g_m)}$$

It can be observed from Figs. 29 and 30 that when R_{ee} is increased from zero, the second order distortion decreases, and the zero point of the third order distortion moves to lower values of $R_e g_m$. When the common mode impedance reaches the value $R_{ee} = (4g_m)^{-1}$, it is no longer possible to get cancellation by increasing R_e. Of course, the impedance R_e still has its linearizing effect.

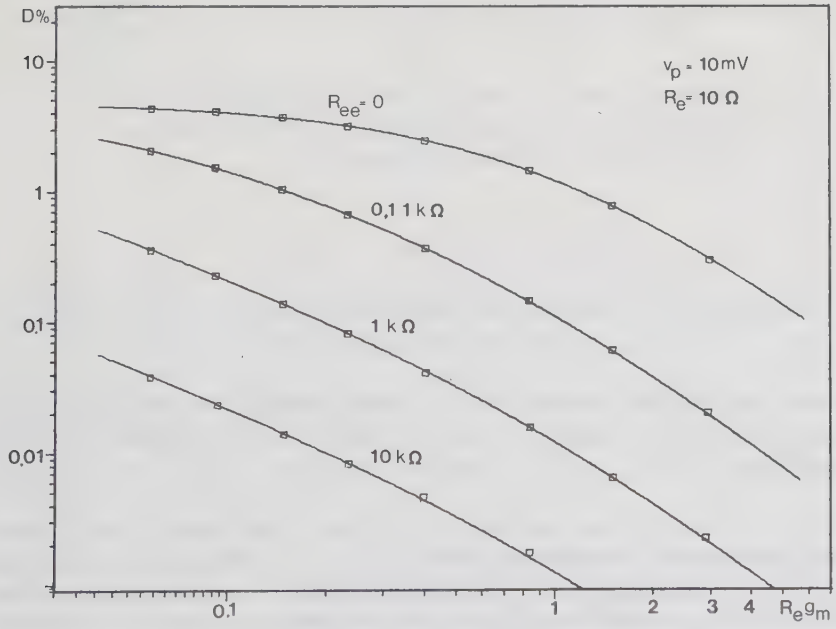

Fig.29 Second order distortion in a single ended differential amplifier as a function of normalized feedback $R_e g_m$

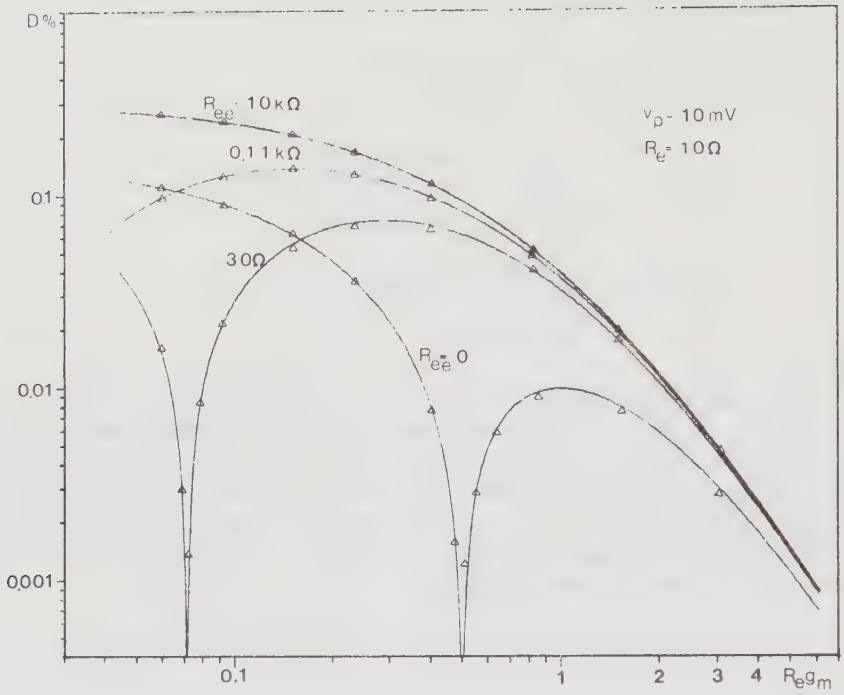

Fig.30 Third order distortion in a single ended differential amplifier as a function of normalized feedback $R_e g_m$.

VI:2 MEASUREMENTS PERTAINING TO THE ICM

The two additional effects of the ICM that will be studied in this section are the Early effect and current gain nonlinearity.

In order to isolate the Early effect the test amplifier should have low generator impedance and high collector load impedance and it should be biased in the low injection region. The test amplifier is outlined in Fig.31. It uses an active collector current source which is paralleled with a variable impedance as collector load. The maximum total load impedance is 900 kΩ shunted with a parasitic capacitance of approximatly 50 pF.

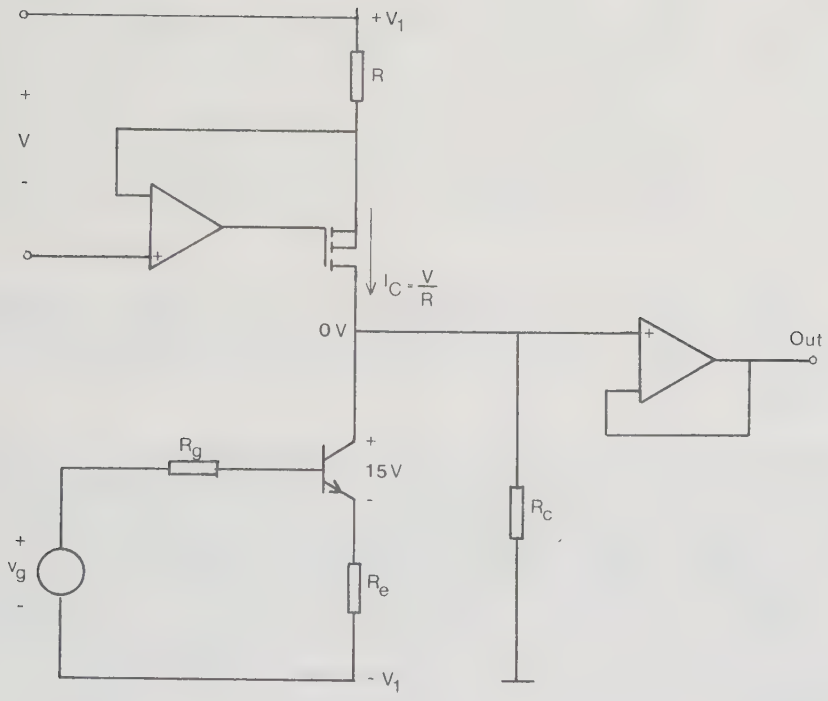

Fig.31 ICM test amplifier

A high input impedance buffer was used to isolate the test amplifier from the analysing equipment.

Since it is essential that the phase shift be kept to a minimum, this measurement series was carried out at a frequency of 100 Hz. The effect of phase shift at the collector is discussed in chapter VII.

The output of the test amplifier was analysed for constant generator voltage and varying collector load impedance. In order to investigate the onset of high injection fenomena, and also the limitation of the derived distortion expression, the collector current was varied in the range 0.1 mA to 8 mA. The obtained data are presented in Figs. 32a to 32f.

The theoretically deduced expression (53) was used to make a curve fit to these data points. The model parameters of chapter V together with the test amplifier impedances were used as inputs to this equation. As can be seen in the diagrams, the parameter V_A, which was considered a constant, had to be modified at higher currents in order to make the theory fit to the measured data. From 0.1 mA up to 1.0 mA the theory agrees with the measured data and shows a constant V_A of 65V. In the range 2mA to 8 mA the value of V_A had to be decreased from 52V to 37V in order to fit with the measured data points. This indicates a current dependent Early voltage in this region [19].

As comparison with the measurement of the output conductance given in chapter V shows that the output conductance in fact increases in this range, making the corresponding value of V_A decrease.

Applying current feedback with an external emitter resistance does not alter the value of the collector load impedance where the points of minimum distortion occure. The emitter resistance has the same linearizing effect as described in chapter VI:1, for collector load impedances below the point of distortion minimum. The diagrams in Figs. 32c to f, shows the effect on distortion of an external emitter resistance of $R_e = 10\,\Omega$ for various currents and as a function of the collector load impedance. The curves were constructed in the same way as described above.

The influence of the effective generator impedance on distortion is shown in Figs. 33 a to d, for the collector current I_C = 1mA . In order to get a measurable output signal as R_g was increased the generator voltage was also increased, and the measured data normalized to v_g = 2mV for comparison reason. The measurements show that increasing the generator impedance will decrease the distortion and lower the walue of load impedance, where cancellation occurs.

For higher currents the effect on distortion of an increase in R_g is more pronounced.Figs. 34 a to c, are similar to Fig.33 but for a collector current I_C = 4 mA.

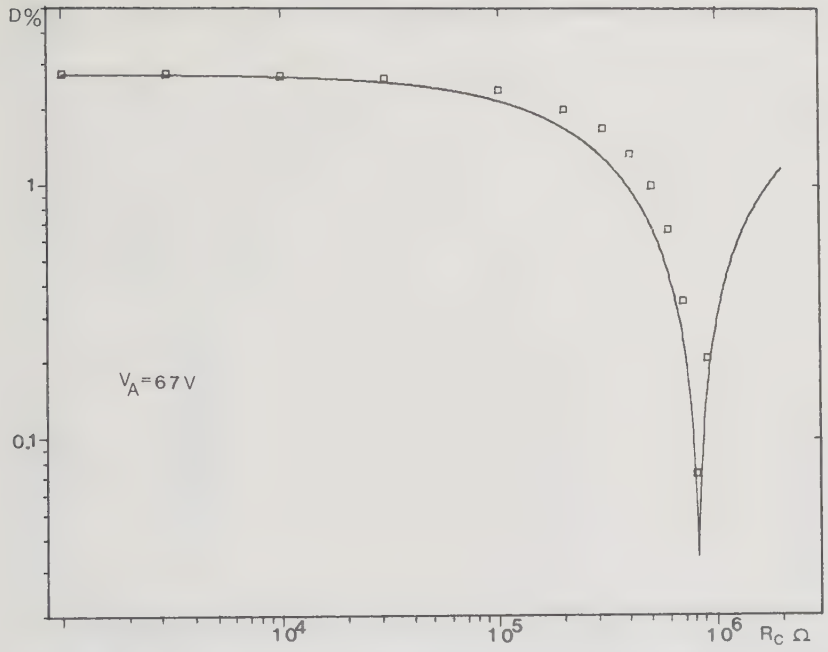

Fig.32a Second order distortion versus R_C for I_C=0.1mA

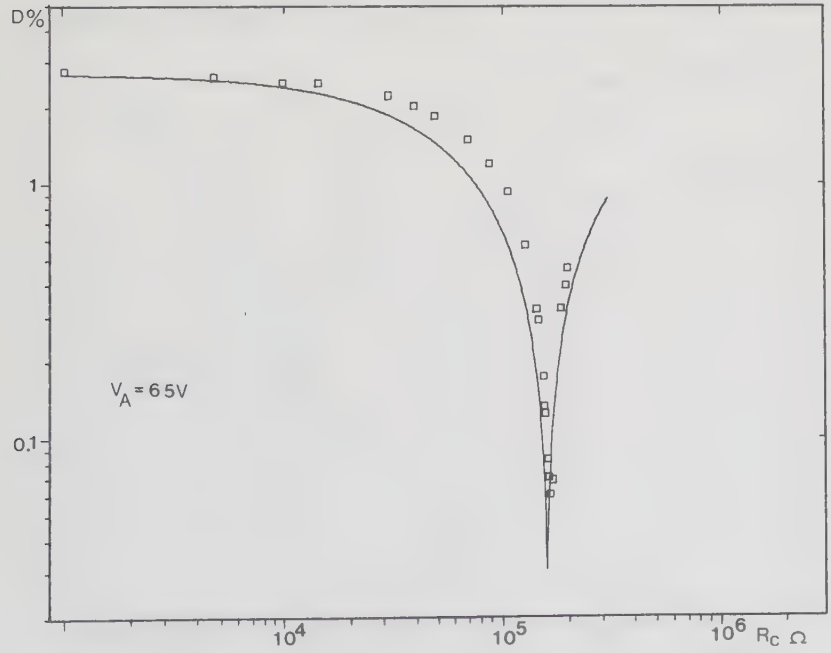

Fig.32b Second order distortion versus R_C for I_C=0.5mA

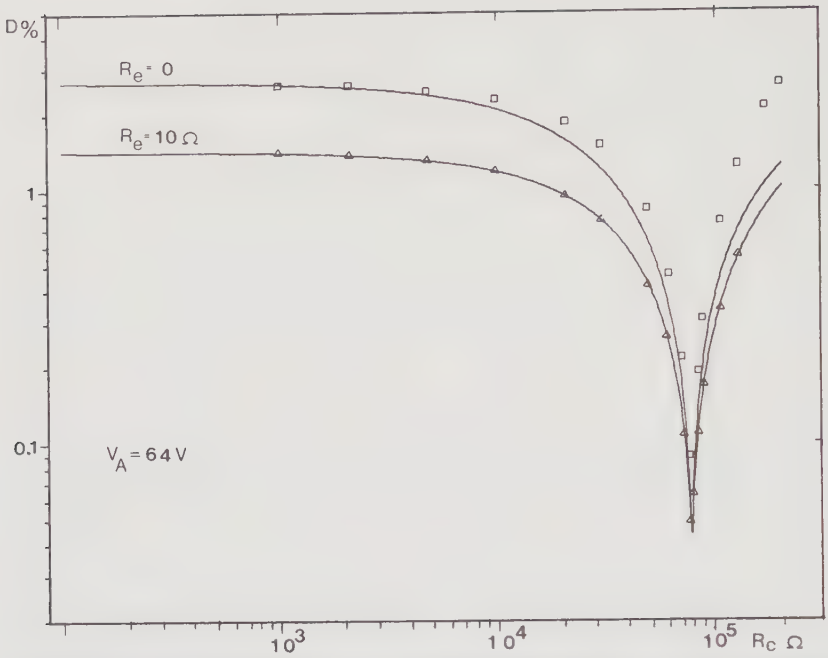

Fig.32c Second order distortion versus R_C for I_C=1.0mA

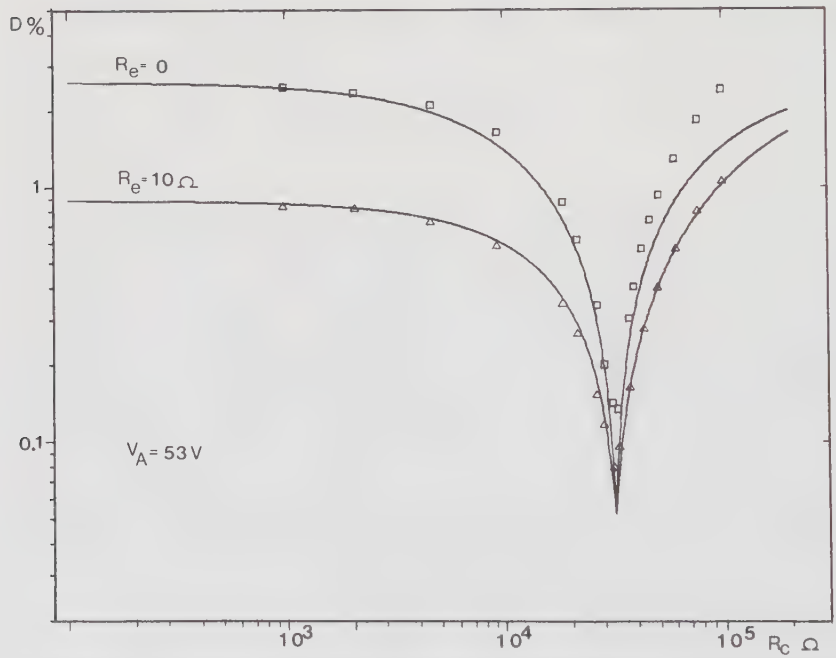

Fig.32d Second order distortion versus R_C for I_C=2.0mA

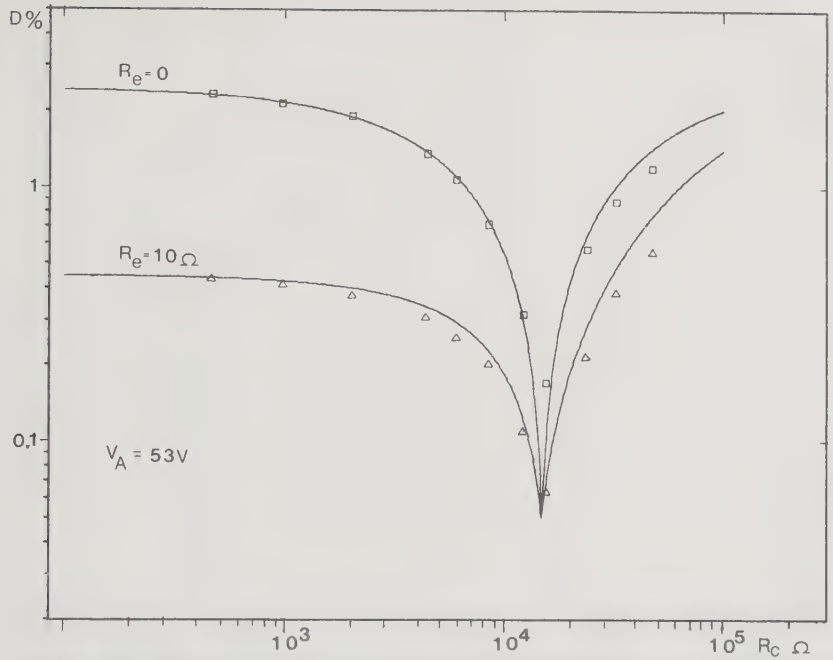

Fig.32e Second order distortion versus R_C for I_C=4.0mA

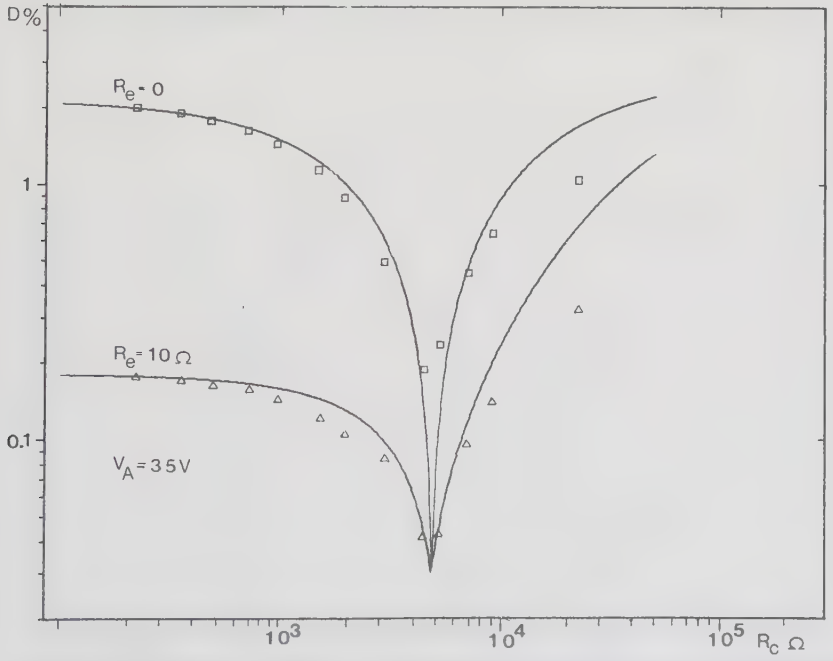

Fig.32f Second order distortion versus R_C for I_C=8.0mA

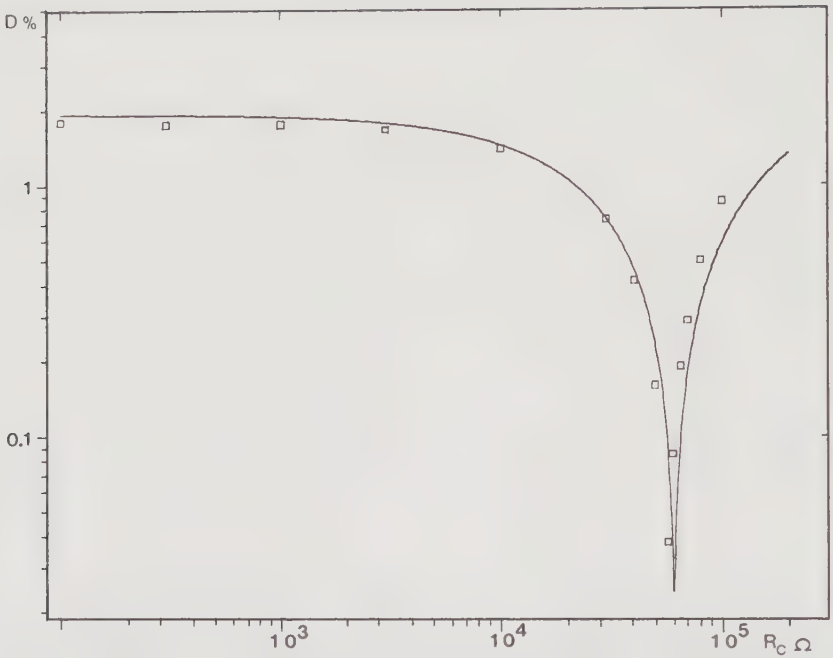

Fig.33a Second order distortion versus R_c for R_g=3k .

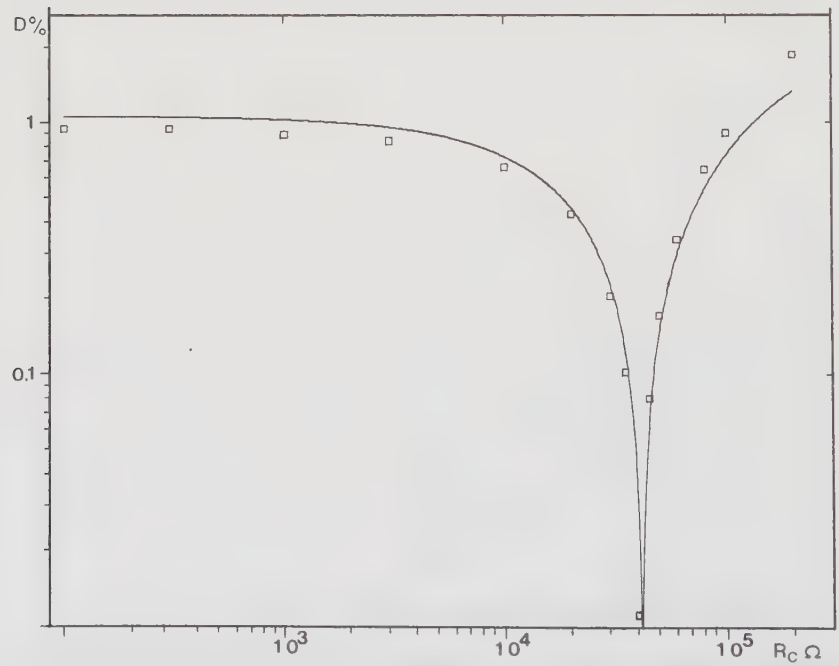

Fig.33b Second order distortion versus R_c for R_g=10k .

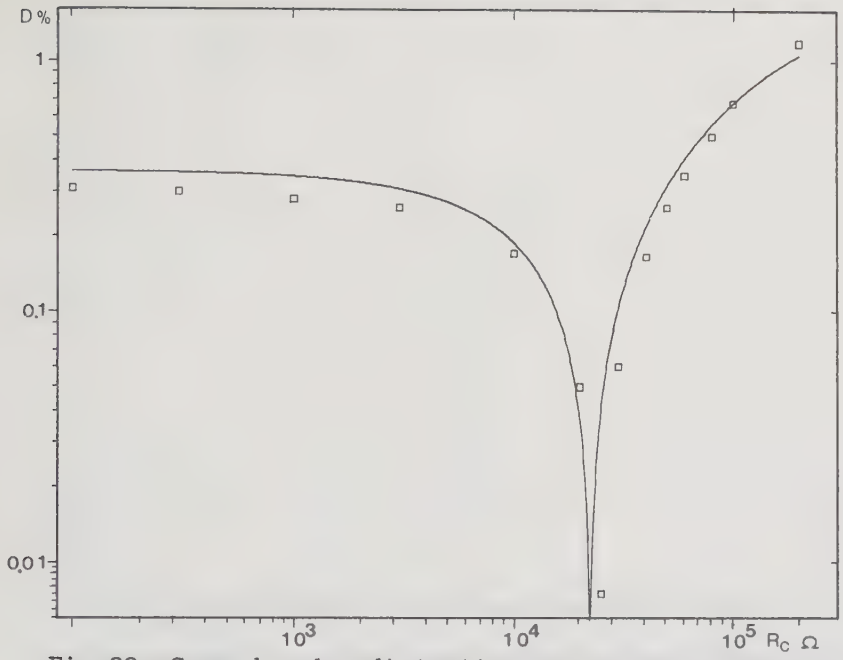

Fig.33c Second order distortion versus R_C for R_g=30k .

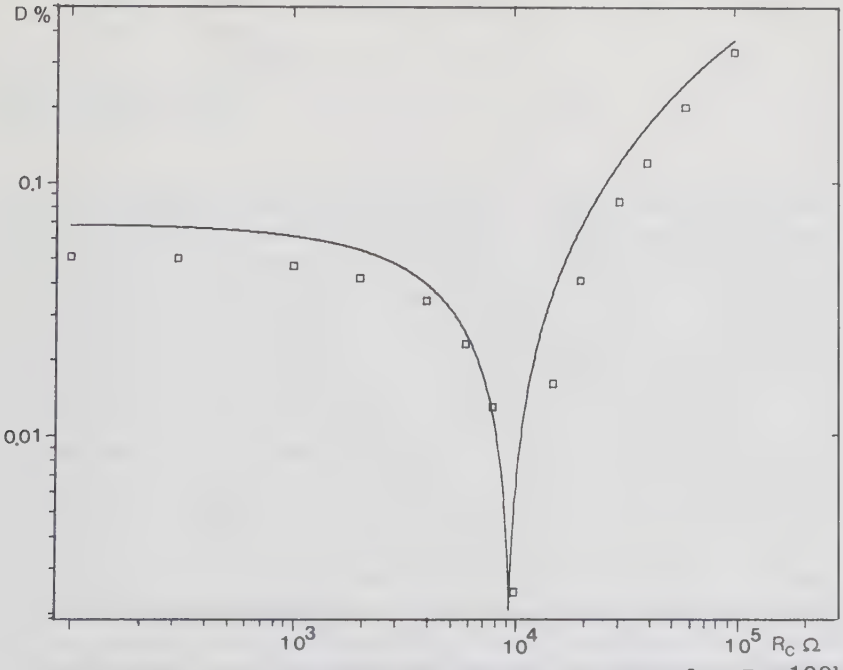

Fig.33d Second order distortion versus R_C for R_g=100k .

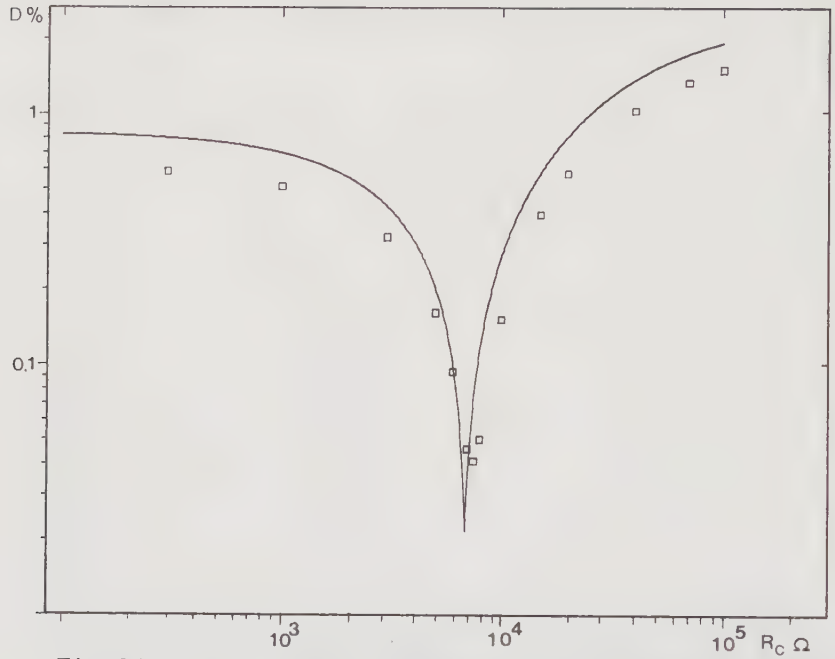

Fig.34a Second order distortion versus R_C for R_g=3k .

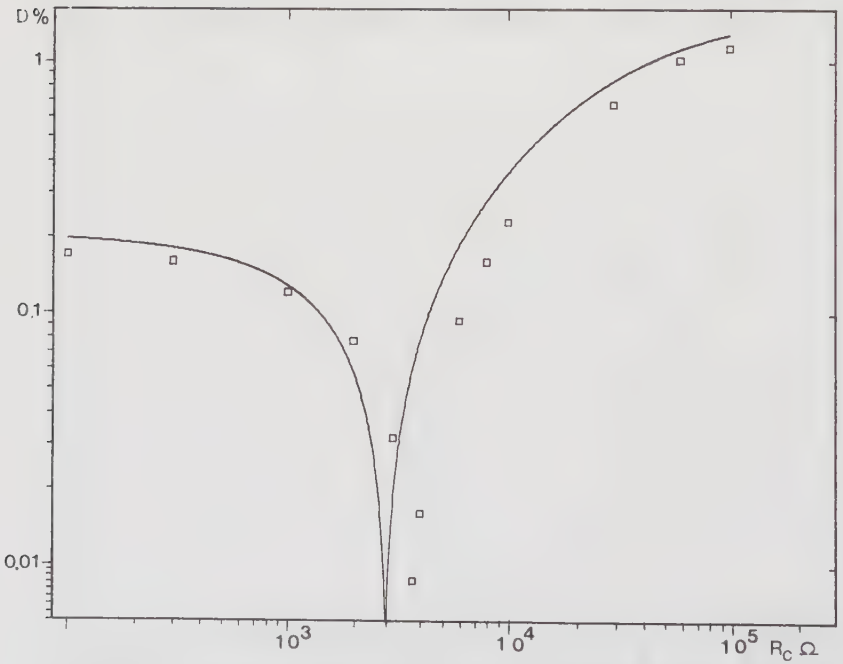

Fig.34b Second order distortion versus R_C for R_g=10k .

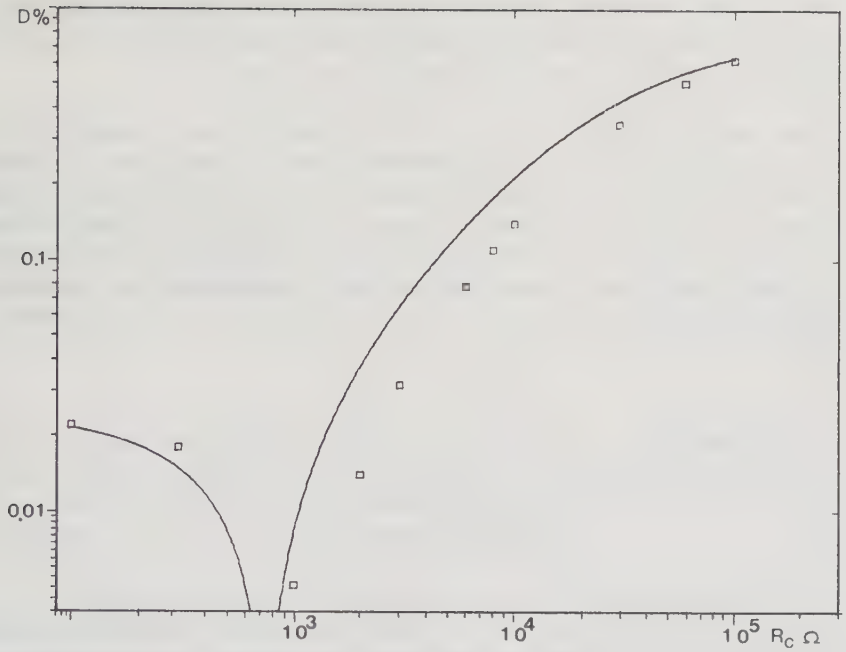

Fig.34c Second order distortion versus R_C for R_g=30k .

VII DISCUSSION

The modification of the ICM used in this text is shown to give accurate modeling of the Early-effect influence on second order distortion. For the transistor BC550C to which the model is applied the variation of distortion as a function of the collector load impedance is predicted in the collector current range 0.1 mA to 1.0 mA . Considering an assumed variation of the Early voltage V_A as a function of current [19], the prediction extends to 8 mA. The influence of the generator impedance is also predicted with accuracy. Due to the very far reaching simplifications made in the derivating process, the accuracy is degraded for regions, where the distortion has changed its phase, i.e. regions where the terms in the numerator of (53) exceed unity. When these terms are of negligible magnitude compared to unity, the equations (53) reduce to the result obtained from the Ebers-Moll model. Thus this could be a way of selecting the proper model complexity.

The effects of reactive components have been omitted in this analysis. However, some remarks on the effect of such components on distortion concellation are due.

Since the cancellation phenomenon is based on feedback from e.g. the collector current to the collector emitter voltage, the phase relation between these two are of significant importance. At the points where second order distortion cancellation occurs, there are current components which have exactly 180 degrees phase shift. Any internal or external reactive component will produce phase shifts that may violate the cancellation.

ACKNOWLEDGEMENTS

To Professor Seth Berglund I wish to express my sincere gratitude for his unreserved support and inspiring discussions during the course of my work.

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I am most grateful to Gunvor Berlin for her readiness and her work of patience at the word processing machine.

Lund May 1982 Clas-Göran Agnvall

APPENDIX I

POWER SERIES REVERSION.

Assume a power series representing a strictly monotonous function:

$$y = \sum a_k x^k \qquad \text{(AI:1)}$$

where the first coefficent a_1 is non-zero. Also assume that the first term is the dominating one for all relevant values of x, or to use the notation of chapter two, the function is weakly nonlinear.

With these assumptions, there exists an inverse of the series AI:1, which we can write [21] :

$$x = \sum b_1 y^1 \qquad \text{(AI:2)}$$

To express the coefficients b_1 in terms of the original ones, we simply substitute AI:1 into AI:2 i.e.

$$x = \sum b_1 (\sum a_k x^k)^1 \qquad \text{(AI:3)}$$

Equating the coefficients of x gives the following system

$$\begin{aligned} & b_1 a_1 = 1 \\ & b_1 a_2 + b_2 a_1^{\,2} = 0 \\ & b_1 a_3 + 2 b_2 a_1 a_2 + b_3 a_1^{\,3} = 0 \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad \text{(AI:4)}$$

Hence the first four coefficients of the reversed series, in terms of the original ones, are:

$$b_1 = \frac{1}{a_1}$$

$$b_2 = -\frac{a_2}{a_1^3}$$

$$b_3 = \frac{2a_2^2 - a_1a_3}{a_1^5} \qquad \text{(AI:5)}$$

$$b_4 = \frac{5a_1a_2a_3 - a_1^2a_4 - 5a_2^3}{a_1^7}$$

As can be observed these expressions grow in complexity with the coefficient indices. Therefore, to keep the necessary number of terms as small as possible, the restriction of weakly nonlinear function is imposed on the original series AI:1.

APPENDIX II

DERIVATION OF DIFFERENTIAL AMPLIFIER TRANSFER FUNCTION.

The labour in deriving the transfer function of this amplifier lies in expressing the sum of the outputs $(y_1 + y_2)$ in (17) in terms of each of the outputs y_1 and y_2, respectively.

As in chapter II:4 the device transfer function can be reversed to give the input x as a function of the output y, i.e.

$$x_1 = \sum b_k y_1^k \qquad x_2 = \sum b_l y_2^l \qquad \text{(AII:1)}$$

Because of the true differential drive, the generator voltage v_g in (17) can be eliminated. Using the result of (AII:1) a relation between the two output variables y_1 and y_2 can be formulated:

$$\sum b_k y_1^k + \beta y_1 + 2\gamma y_1 = -\sum b_l y_2^l - \beta y_2 - 2\gamma y_2 \qquad \text{(AII:2)}$$

This relation can be divided into two separate equations by the definition of an auxiliary variable z i.e.

$$\begin{aligned} z &= \sum b_k y_1^{\,k} + \beta y_1 + 2\gamma y_1 \\ z &= -\sum b_1 y_2^{\,1} - \beta y_2 - 2\gamma y_2 \end{aligned} \qquad \text{(AII:3)}$$

Reversing the equations A:3 gives the outputs y_1 and y_2 in terms of the auxiliary variable z as:

$$\begin{aligned} y_1 &= \sum c_k z^k \\ y_2 &= \sum c_1 (-z)^1 \end{aligned} \qquad \text{(AII:4)}$$

The sum of the two outputs can be formed, neglecting the fourth and higher order terms, as:

$$y_1 + y_2 = 2c_2 z^2 \qquad \text{(AII:5)}$$

where the coefficient c_2 is found to be:

$$c_2 = -\frac{b_2}{(b_1 + \beta + 2\gamma)^3} \qquad \text{(AII:6)}$$

From eq A:3 an expression of z^2 in terms of either y_1 or y_2 can be found. For simplicity the y_2 case is left out in the following derivation because it differs from the y_1 case only in sign. For the y_1 case we can write:

$$\begin{aligned} z^2 &= [(b_1 + \beta + 2\gamma)y_1 + b_2 y_1^{\,2} + \ldots]^2 = \qquad \text{(AII:7)} \\ &= (b_1 + \beta + 2\gamma)^2 y_1^{\,2} + 2b_2(b_1 + \beta + 2\gamma)y_1^{\,3} + \ldots \end{aligned}$$

Substituting equations AII:6 and AII:7 into AII:5 yields:

$$y_1 + y_2 = -\frac{2b_2}{(b_1 + \beta + 2\gamma)} y_1^{\,2} - \frac{4b_2^{\,2}}{(b_1 + \beta + 2\gamma)^2} y_1^{\,3} + \ldots \qquad \text{(AII:8)}$$

This expression of (y_1+y_2) is fed into the input equations (17), together with (AII:1), to give the generator voltage v_g in terms of the output y_1:

$$x_g/2 = (b_1 + \beta)y_1 + \frac{b_2(b_1 + \beta)}{(b_1 + \beta + 2\gamma)} y_1^2 +$$

$$+ [b_3 - \frac{4b_2^2}{(b_1 + \beta + 2\gamma)^2}]y_1^3 + .. \qquad \text{(AII:9)}$$

Reversing this equation we arrive at:

$$y_1 = \frac{1}{(b_1 + \beta)}(x_g/2) - \frac{b_2}{(b_1 + \beta)^2(b_1 + \beta + 2\gamma)}(x_g/2)^2 +$$

$$+ \frac{2b_2^2}{(b_1 + \beta + 2)(b_1 + \beta)^4} - \frac{b_3}{(b_1 + \beta)^4} (x_g/2)^3 + \ldots \qquad \text{(AII:10)}$$

Finally, substituting the coefficients b_k of the inverse device transfer function of (AII:1) into (AII:10), gives us the total transfer function:

$$y_1 = d_1(x_g/2) + d_2(x_g/2)^2 + d_3(x_g/2)^3 + \ldots$$
$$y_2 = -d_1(x_g/2) + d_2(x_g/2)^2 - d_3(x_g/2)^3 + \ldots \qquad \text{(AII:11)}$$

where:

$$d_1 = \frac{a_1}{(1 + \beta a_1)} \qquad d_2 = \frac{a_2}{\lambda(1 + \beta a_1)^3} \qquad d_3 = \frac{a_3 - 2a_2^2\delta/a_1}{(1 + \beta a_1)^4}$$

$$\lambda = \frac{1 + a_1 + 2a_1}{(1 + \beta a_1)} \qquad \delta = \frac{\beta a_1 + 2\gamma a_1}{(1 + \beta a_1 + 2\gamma a_1)}$$

APPENDIX III

DERIVING THE DISTORTION EXPRESSION USING THE MODIFIED ICM

From (58) we have for the collector current:

$$I_C = I_S \exp(V_{BE}/V_T)(1 - q_{jC} - I_C/I_K) \qquad \text{(AIII:1)}$$

Using the low voltage approximation for the Early-effect from (51), one can write:

$$(1 - q_{jC} - I_C/I_K) = [1 - q(I_C)] \qquad \text{(AIII:2)}$$

where: $$q = - V_{CE}/V_A + I_C/I_K$$

The base-emitter voltage with the sign convention of Fig. 23 is given by:

$$V_{BE} = V_G - (R_b + R_c)I_B - R_e I_C \qquad \text{(AIII:3)}$$

To facilitate the derivation process we can write the collector current as a product of two functions

$$I_C(V_G) = E(V_G, I_C)(1 - q(I_C)) \qquad \text{(AIII:4)}$$

Denoting the partial derivatives with indices as follows:

$dE/dV_G = E_V'$; $dI_C/dV_G = I_V'$; $d^2E/dI_C dV_G = E_{IV}''$ and so on.

For the first derivative we have:

$$I_V' = (E_V' + E_I' I_V')(1 - q) + E(-q_I' I_V') \qquad \text{(AIII:5)}$$

where:

$$E_V' = I_C/[V_T(1-q)]$$

$$E_I' = -I_C R_{eq}/[V_T(1-q)]$$

$$R_{eq} = (R_b + R_e)dI_B/dI_C + R_e$$

$$q_I' = (R_c + R_e)/V_A + 1/I_K$$

Thus the transfer gain can be written:

$$dI_C/dV_G = \frac{I_C}{V_T[1 + I_C R_{eq}/V_T + I_C q'/(1-q)]} \qquad \text{(AIII:6)}$$

Continuing the derivation under the assumption that $q'' = 0$ we have after some rearrangements:

$$I_{VV}''[1 - E_I'(1-q) + Eq_I'] = E_{VV}''(1-q) + 2I_V'[E_{IV}''(1-q) - E_V' q_I'] + \\ + I_V^2[E_{II}''(1-q) - 2E_I' q_I'] \qquad \text{(AIII:7)}$$

where:

$$E_{VV}'' = I_C/[V_T^2(1-q)]$$

$$E_{IV}'' = -I_C R_{eq}/[V_T^2(1-q)]$$

$$E_{II}'' = I_C R_{eq}^2/[V_T^2(1-q)] - I_C R_{eq}'/[V_T(1-q)]$$

$$R_{eq}' = dR_{eq}/dI_C = (R_b + R_e)d^2 I_B/dI_C^2$$

Resulting in the expression:

$$d^2 I_C/dV_G^2 = \frac{I_C\{1 - [I_C q'/(1-q)]^2 - I_C^2 R_{eq}'/V_T\}}{V_T^2[1 + I_C q'/(1-q) + I_C R_{eq}/V_T]^3} \qquad \text{(AIII:8)}$$

For sinusoidal inputs we have:

$$v_g = v_p \sin \omega t$$

and this gives us the total collector current as:

$$I_C + i_c = I_C + dI_C/dV_G(v_p \sin\omega t) + 1/2 d^2 I_C/dV_G^2 (v_p \sin\omega t)^2 + \ldots$$

Making use of the equation (7) and (12) we have for the harmonic distortion:

$$D_2 = 1/4 v_p (d^2 I_C/dV_G^2)(dI_C/dV_G)^{-1}$$

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